Time Domain Multiconductor Transmission Line Analysis Using Effective Internal Impedance

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Frequency domain approach: using effective internal impedance (EII)

- not the same as standard surface impedance boundary condition (SIBC)
- inside of conducting material is replaced with external medium
- sheet current: produce original external field and non-zero inside field
- sheet impedance
  - “Effective Internal Impedance”
  - hard to calculate exactly
  - can be approximated
Approximation of EII using transmission line model

- no unique approximation
- decompose bar into plate and triangular sections: effectively capture current crowding near the edge
- $Z_{eii}$ of each geometry determined by calculating input impedance

\[
\begin{align*}
\text{Restrictions:} & \\
\mathcal{R}(Z_{eii})_{f \to 0} &= \frac{1}{\sigma A} \\
\mathcal{R}(Z_{eii})_{f \to \infty} &= \frac{1}{\sigma \delta w} (1 + j) \\
Z_{eii}^{\text{plate}} &= \frac{(1 + j)/(\sigma \delta)}{\tanh[(1 + j)t/(2 \delta)]} \cdot \frac{1}{W} \\
Z_{eii}^{\text{triangle}} &= \frac{j(1 + j) J_0(j(1 + j)h/\delta)}{\sigma \delta} \frac{1}{J_1(j(1 + j)h/\delta)} \cdot \frac{1}{w}
\end{align*}
\]
Surface ribbon method (SRM) in frequency domain

- \[ Z_{eii} [I] + s[L][I] = -\frac{\partial}{\partial z} [V] \]

- EII assigned to each ‘ribbon’
- mutuals between ribbons: capture external behaviors
- efficiency of SRM:
  - matrix size scales with conductor perimeter
  - minimum segmentation method

Unknowns: MSM-8, SRM-52, VFM-420
Effects of the frequency dependencies on time domain waveform

- line 1 excited with 0.1ns rise and fall time
- measured far end of line 4 (length=0.1 m)
- $R_s = 5 \text{ Ohm}$, $C_L = 10 \text{ pF}$
Equivalent circuit modeling for EII: transforming frequency domain EII into time domain

- each ‘isolated conductor’ cross-sections divided into 4 parts
- each part represented with 1 resistor and 1 inductor
- rules to determine values of circuit elements

\[ \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = RR \quad \frac{L_1}{L_2} = \frac{L_2}{L_3} = LL \]

- additional constraints: correct DC resistance and inductance

- RR and LL are empirically determined constants unique to the geometry of the conductor

\[ Z_{eii}(s) = R_1 \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]
Time domain conversion using equivalent circuit model

- equivalent circuit model
  - can be easily constructed
  - rational function in s-domain, exponential function in time domain
  - problem size can be reduced using Padé approximation: dominant pole reduction
  - time domain convolution problem can be solved using recursive properties

blue: numerical result
red: circuit model
Derivation of time domain equation

- frequency domain (s-domain) equation
  \[
  \left[ \frac{Z_{ei}}{s} \right] s[l] + [L] s[l] = -\frac{\partial}{\partial z} [V]
  \]

- transformation into time domain
  \[
  L^{-1}\left( \left[ \frac{Z_{ei}}{s} \right] \right) = \left[ R_i \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s(a_3 s^3 + a_2 s^2 + a_1 s + a_0)} \right] \\
  = [R_i \sum K_i e^{p_i t}] = [\zeta(t)]
  \]

  \[
  [\zeta(t)] \ast \frac{\partial}{\partial t} [l] + [L] \frac{\partial}{\partial t} [l] = -\frac{\partial}{\partial z} [V]
  \]

- time domain convolution
  \[
  Y(n\Delta t) = X(n\Delta t) \ast K e^{p(n\Delta t)}
  \]

  \[
  = K\Delta t \cdot X(n\Delta t) + e^{p\Delta t} \cdot Y((n - 1)\Delta t)
  \]

- application of recursive equation
  \[
  [K] \frac{\partial}{\partial t} [l] + [L] \frac{\partial}{\partial t} [l] + [V_{ds}] = -\frac{\partial}{\partial z} [V]
  \]

  \[
  [L'] \frac{\partial}{\partial t} [l] + [V_{ds}] = -\frac{\partial}{\partial z} [V]
  \]

  \[
  [C] \frac{\partial}{\partial t} [V] + [G] \cdot [V] = -\frac{\partial}{\partial z} [l]
  \]

- lossless like equation with additional voltage source

- voltage source depends on poles, residues, time step, and values from previous time step

- different simulators can be used to solve equations
Example I: time stepping solution (FDTD)

- make finite difference approximation to the partial derivatives: extra current source compared to lossless case
- each voltage and adjacent current solution point separated by $\Delta z/2$
- $\Delta t$ has to be kept small to satisfy stability condition: may not be appropriate for electrically long lines

\[
\begin{align*}
[I]_{k+\frac{1}{2}}^{n+\frac{1}{2}} &= [I]_{k+\frac{1}{2}}^{n+\frac{1}{2}} - \left[ \frac{L'}{\Delta t} \right]^{-1} \left( \frac{[V]_{k+1}^{n+1} - [V]_{k}^{n+1}}{\Delta z} \right) + [V_{ds}]_{k}^{n+1} \\
&= [I]_{k+\frac{1}{2}}^{n+\frac{1}{2}} - \left[ \frac{L'}{\Delta t} \right]^{-1} \left( \frac{[V]_{k+1}^{n+1} - [V]_{k}^{n+1}}{\Delta z} \right) - [I_{ds}]_{k}^{n+1} \\
[V]_{k}^{n+1} &= [V]_{k}^{n} - \left[ \frac{C}{\Delta t} \right]^{-1} \left( \frac{[I]_{k+\frac{1}{2}}^{n+\frac{1}{2}} - [I]_{k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} \right)
\end{align*}
\]
Example I: continued

- line 1 excited with 0.1 ns rise and fall time trapezoidal waveform
- line length = 0.1m, $R_S = 5$ Ohm, $R_L = 50$ Ohm
Using EII for time domain simulation

- frequency domain concept (SRM) can be easily applied to time domain using equivalent circuit for EII
- excellent agreement with FFT calculations
- significant decrease in run-times demonstrated
- computation time can be further reduced: run time can be comparable to simple R-L-C circuit analysis
  - dominant pole approximation
  - minimum segmentation
- can easily include dc-to-skin effect behavior directly in time domain models