A New, Accurate Quasi-Static Model for Conductor Loss in Coplanar Waveguide

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Conductor loss calculation techniques

- Simple dc+skin depth resistance model
- Incremental inductance rule
  - Numerical evaluation (Wheeler)
  - Analytical evaluation (Gupta)
- Conformal mapping (Collin)
- Closed form expression (e.g., W. Heinrich, Trans. MTT, Jan.'93)
- Fullwave calculations
New Quasi-Static Technique

- Conformal mapping used to find frequency dependent series resistance and inductance, in addition to normal capacitance per unit length.

- Requires the use of a "scaled" conductivity in mapped domain:
  - surface impedance can be calculated using scaled conductivity.
  - Surface impedance can be found in real space, "scaled" in mapped domain.
Length Scaling Under the Influence of a Conformal Map

- Conformal maps "preserve" local shape
  - differential lengths in $w(u, v)$ plane are "scaled" by an amount $M(u, v)$

$$\frac{dx}{du} = M \frac{dy}{dv} = M$$

$$w(u, v) = f(z)$$

$$M(u, v) = \left| \frac{df}{dz} \right|_{u,v}$$
Example of scaling for a simple cylindrical wire

\[ \sigma \neq 0 \quad \sigma = 0 \quad \sigma_M \neq 0 \quad \sigma_M = 0 \]

Conformal map
• Shunt capacitance depends on the ratio of side lengths:

\[
\partial C = \frac{r \partial \theta}{\partial r} = \frac{(\partial v/M)}{(\partial u/M)} = \frac{\partial v}{\partial u}
\]

• Scale factor cancels!

• Series resistance depends on the area:

\[
\partial R = \frac{1}{\sigma} \frac{1}{r \partial r \partial \theta} = \frac{1}{\sigma} \frac{1}{(\partial u/M)} \frac{1}{(\partial v/M)} = \left( \frac{M^2}{\sigma} \right) \frac{1}{\partial u \partial v}
\]

\[
\therefore \sigma_M (scaled) = \frac{\sigma}{M^2}
\]

• Scale factor enters as \(M^2\)!
Conductor Surface Impedance of a Circular Cylindrical Wire

• Can solve Helmholtz equation exactly

\[ Z_s = - \frac{T J_o(T r_o)}{2 \pi r_o \sigma J'_o(T r_o)} \]

where, \( T^2 = -j \omega \mu \sigma \)

• In mapped domain requires solution for conducting rectangular slab with non-uniform conductivity
  - can be solved using non-uniform transmission line equations

• For both real & mapped planes, analytic expressions for \( Z_{surf} \) are identical
Conformal mapping & "scaling" in Co-planar waveguide

- Mapping is performed by evaluating elliptic integral of first kind
- Surface impedance is "scaled" in the mapped domain to include effect of current crowding
- Surface resistance is approximated as

\[
R_s(\omega) = \text{Re}\left[\frac{\sqrt{\frac{\omega \mu_0}{2\sigma}}}{2 \tanh}\left(\frac{t}{2}\right) (1 + j)\right]
\]
Diagram illustrating use of a conformal map to find the series impedance of a transmission line including the effect of finite resistance.
Series impedance $Z$ due to differential width $du$ is the series combination of mapped surface resistance and parallel plate inductance:

$$\partial Z = \frac{R_S \left\{ M(u, v_t) + M(u, v_b) \right\}}{du} + \frac{j\omega \mu_0 |v_t - v_b|}{du}$$

Therefore, total series impedance per unit length is

$$Z(\omega) = \left[ \int_0^{u_0} \left\{ \frac{du}{j\omega \mu_0 |v_t - v_b| + R_S \left\{ M(u, v_t) + M(u, v_b) \right\}} \right\} \right]^{-1}$$
At low frequency total series impedance reduces to

\[ Z = R_{dc} + j \omega \mu_0 |v_t - v_b| \left( \frac{R_{dc}}{R_s} \right)^2 \int_0^{u_0} du \left[ M(u, v_t) + M(u, v_b) \right]^2 + j \omega L_{ext} \]

At high frequency
Cross-sectional view of CPW. Dimensions used for SI GaAs and Pyrex samples are $a = 5 \, \mu m$, $b = 12 \, \mu m$, $w = 500 \, \mu m$
Comparison with our model: SI GaAs substrate

![Comparison Graphs]

**GaAs substrate**

- **Attenuation (dB/cm)**
  - Frequency (GHz): 0.01, 0.1, 1, 10, 100
  - Attenuation: 0, 1, 2, 3, 4, 5, 6, 7

- **Effective index of refraction**
  - Frequency (GHz): 0.01, 0.1, 1, 10, 100
  - Effective index of refraction: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Equivalent series resistance and inductance: GaAs substrate

Series Resistance (ohms/cm)

Series Inductance (nH/cm)

Frequency (GHz)

0.04 0.1 0.5 1 10 100

0 1 2 3 4 5 6 7

0 20 40 60 80 100 120

GaAs substrate

inductance

resistance
Comparison with our model: Pyrex substrate

Attenuation (dB/cm) vs Frequency (GHz)

Effective index of refraction vs Frequency (GHz)
Equivalent series resistance and inductance: Pyrex substrate

Series Resistance (Ohms/cm)

Series Inductance (nH/cm)

Frequency (GHz)

Pyrex substrate
Comparison with other techniques
(GaAs substrate)

Measured
Rdc+skin effect

Wheeler

Attenuation (dB/cm)
Attenuation (dB/cm)

Frequency (GHz)
Frequency (GHz)
Comparison with other techniques (GaAs substrate)

![Graph showing comparison of attenuation vs. frequency for Measured, Gupta, and Collin with GaAs substrate.](image-url)
Comparison with Haydl's experimental data

InP substrate
InP

- **Yellow Square**: $a = 2 \, \mu m$, $b = 15 \, \mu m$, $t = 0.25 \, \mu m$
- **Green Triangle**: $a = 2 \, \mu m$, $b = 15 \, \mu m$, $t = 0.5 \, \mu m$
- **Red Diamond**: $a = 6 \, \mu m$, $b = 45 \, \mu m$, $t = 0.25 \, \mu m$
- **White Circle**: $a = 6 \, \mu m$, $b = 45 \, \mu m$, $t = 0.5 \, \mu m$
InP

- \( a = 11 \, \mu m, b = 15 \, \mu m, t = 0.25 \, \mu m \)
- \( a = 11 \, \mu m, b = 15 \, \mu m, t = 0.5 \, \mu m \)
- \( a = 33 \, \mu m, b = 45 \, \mu m, t = 0.25 \, \mu m \)
- \( a = 33 \, \mu m, b = 45 \, \mu m, t = 0.5 \, \mu m \)
Conclusions

• Conformal mapping can be used to accurately predict conductor loss in quasi-TEM transmission lines
  - can be applied to coplanar strips, microstrip, strip-line

• Numerically efficient

• Closed form

• Easy to implement in CAD software

• Useful for interconnect loss calculation or timing analysis in Multi Chip Modules (MCMs)
  - generates equivalent circuit model that is very efficient for time domain simulation