

# Highly Accurate Quasi-Static Modeling of Microstrip Lines Over Lossy Substrates

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## **Abstract**

A highly accurate, analytic quasi-static model of a microstrip over a semiconductor layer has been developed. The model agrees with full-wave calculations in all three modes of propagation (skin-effect, slow wave, and dielectric quasi-TEM), for both the attenuation constant  $\alpha$  and the propagation constant  $\beta$  over a very wide range of dimension, substrate conductivity, and frequency. To achieve this level of agreement, a non-uniform cross-section, transverse resonance technique has been applied to find the series impedance per unit length of the microstrip transmission line.

## **I. Introduction**

There has been a great deal of interest in modeling microstrip transmission lines on semiconducting substrates, including more recent interest in the use of silicon as a microwave substrate. Interconnects fabricated on multi-layered semiconductor substrates (such as silicon dioxide on silicon) produce behavior that is more difficult to predict than that of lines made on lossless substrates [1, 2]. In 1971 Hasegawa *et al.* [3] experimentally verified this behavior for a microstrip on an SiO<sub>2</sub>-Si substrate. Here we show that quasi-static analysis can accurately predict

the behavior of such transmission lines, with excellent agreement between full-wave and static model over a very wide range of dimension, substrate conductivity, and frequency.

To evaluate the impact of a semiconductor layer of conductivity  $\sigma$  on the transmission line changes in both electric and magnetic fields must be determined. For a microstrip-like geometry (Figure 1) the changes in the electric field are relatively straightforward. If the frequency of the applied signal is below the dielectric relaxation frequency of the semiconductor  $\sigma/\epsilon_{semi}$ , the electric fields behave as if the semiconductor were a metallic sheet. Conversely, if the frequency is increased or conductivity decreased until  $\omega > \sigma/\epsilon_{semi}$ , the electric fields behave as if the semiconductor were a lossy dielectric layer. In the crossover region where  $\omega \sim \sigma/\epsilon_{semi}$ , the impact of the semiconductor conductivity on propagation loss can be very large [4].

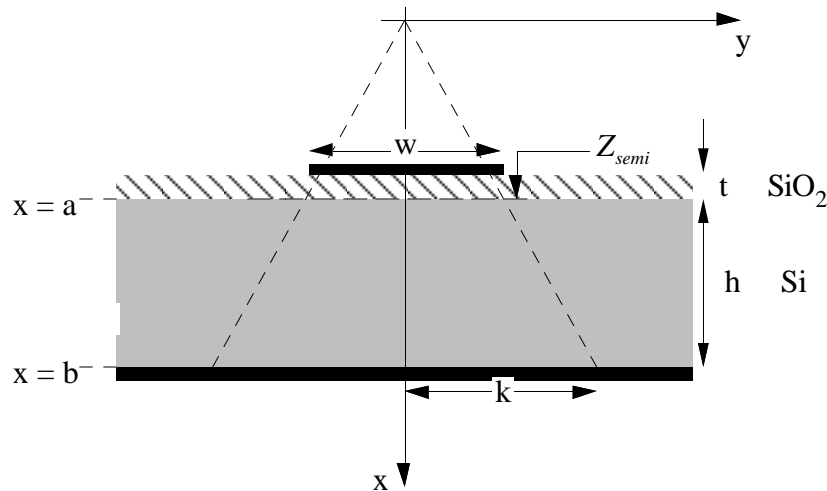


Figure 1: Cross section of a microstrip on an oxide-semiconductor substrate.

The proper value of series inductance for the transmission line must also be determined. When the thickness of the semiconducting substrate becomes greater than the skin depth, the so-called "skin-effect" mode of propagation is encountered [3]. Several previous papers have recognized that this leads to a reduction in the effective separation between the signal and the ground plane, as well as inducing significant loss due to series resistance [3, 5]. In contrast, if the frequency or conductivity is low enough that the skin depth is larger than the thickness of the semiconductor, the magnetic fields (and thus inductance  $L$ ) will be determined primarily by the separation of the microstrip and the true ground plane.

## II. Quasi-Static Model

Many quasi-static models have been proposed for microstrips over semiconducting layers that adequately describe the impact of finite conductivity on the shunt admittance per unit length  $Y$  of the transmission line [6]. The equivalent circuit used consists of a capacitor  $C_{insu}$ , representing the top dielectric layer, in series with a parallel capacitance  $C_{semi}$  and conductance  $G_{semi}$ , representing the semiconducting layer. For simplicity we assume that the top dielectric layer thickness  $t$  is much less than the microstrip width  $w$ , so

$$C_{insu} = \frac{\epsilon_{insu}}{t} w \quad (1)$$

where  $\epsilon_{insu}$  is the dielectric constant of the top insulating layer. For the semiconducting layer, the shunt conductance  $G_{semi}$  scales identically with its capacitance. Here, we use Wheeler's equations [7] to find the quasi-static capacitance due to the semiconductor portion of the interconnect,  $C_{semi}$ , and then the conductance is

$$G_{semi} = \frac{\sigma}{\epsilon_{semi}} C_{semi} \quad (2)$$

where  $\epsilon_{semi}$  is the dielectric constant of the semiconductor. The use of Wheeler's equations efficiently accounts for thickness variations of the semiconductor layer  $h$  with respect to the microstrip width. Thus, the total admittance per unit length for the interconnect is given by

$$Y = \frac{j\omega C_{insu} G_{semi} - \omega^2 C_{semi} C_{insu}}{G_{semi} + j\omega(C_{semi} + C_{insu})} \quad (3)$$

The semiconductor layer can also significantly affect the series impedance per unit length  $Z$  of the microstrip. In general this effect has not been adequately treated in previous quasi-static models. Here we use the transverse resonance technique to find the surface impedance of the ground plane as seen through the semiconductor layer, similar to that previously used by [5]. Previous work assumed that the equivalent transverse transmission line is of uniform cross section with a short circuit boundary condition representing the perfect ground plane. A uniform cross section approximation, however, is invalid for microstrip except for very wide strips (i.e.,  $w \gg h$ ). If the effective cross section is assumed to vary linearly with depth  $x$  (Fig. 1), approximating

the spreading of the fields between the microstrip and the ground plane, the input impedance of this non-uniform transmission line is the desired surface impedance, and is given by

$$Z_{semi} = \frac{1}{j\omega} \sqrt{\frac{j\omega\mu_0}{j\omega\epsilon_{semi} + \sigma}} \frac{\mathbf{H}_0^{(2)}(j\beta_s b)\mathbf{H}_0^{(1)}(j\beta_s a) - \mathbf{H}_0^{(2)}(j\beta_s a)\mathbf{H}_0^{(1)}(j\beta_s b)}{\mathbf{H}_0^{(2)}(j\beta_s b)\mathbf{H}_1^{(1)}(j\beta_s a) - \mathbf{H}_1^{(2)}(j\beta_s a)\mathbf{H}_0^{(1)}(j\beta_s b)} \quad (4)$$

where  $\mathbf{H}_n^{(1)}$  and  $\mathbf{H}_n^{(2)}$  are Hankel functions of the first and second kind,  $\beta_s = \sqrt{j\omega\mu_0(j\omega\epsilon_{semi} + \sigma)}$ ,  $a = \frac{hw}{2k - w}$ , and  $b = a + h$ . The distance  $k$  is a measure of how much the fields spread before

reaching the ground plane. The total impedance per unit length for the microstrip is then

$$Z = Z_i \frac{Z_{semi} + Z_i \tanh(\gamma_i t)}{Z_i + Z_{semi} \tanh(\gamma_i t)} \quad (5)$$

where  $Z_i = \sqrt{\mu_0 / \epsilon_{insu}} / w$  and  $\gamma_i = j\omega\sqrt{\mu_0\epsilon_{insu}}$ . In the limit of zero conductivity in the semiconductor, the impedance calculated using eq. 5 should reduce to the inductance of a simple microstrip line. For  $k = 3h + w/2$ , the inductance calculated using eq. 5 matches that obtained from Wheeler's equations [7] very closely (within 3 %) over a wide range of  $h/w$  (height-width ratio range of at least  $10^{-2}$  to  $10^2$ ), as shown in Fig. 2 below.

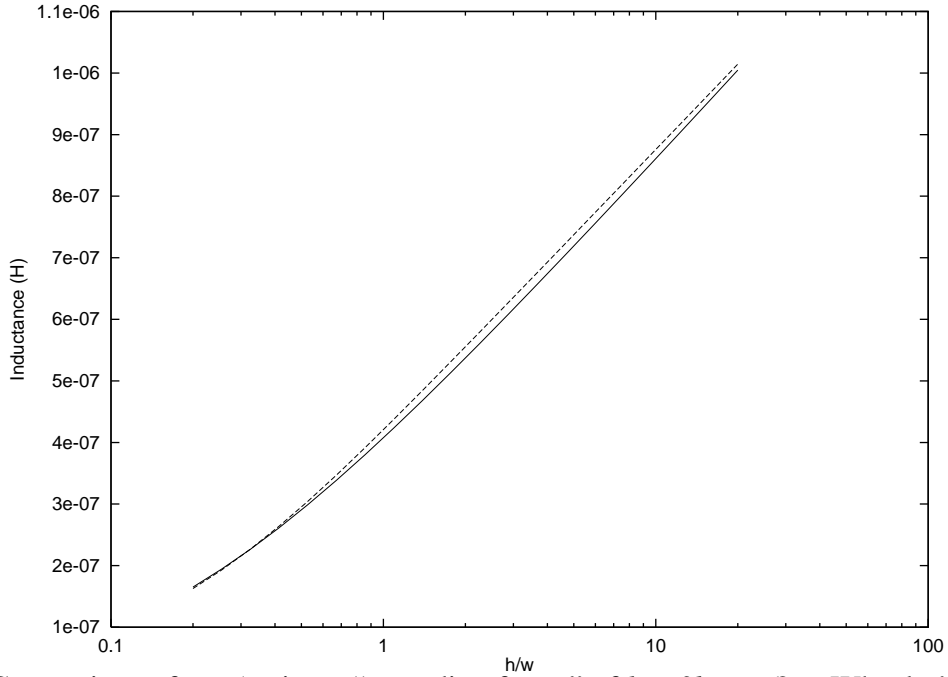


Figure 2: Comparison of eq. 5 using a “spreading factor” of  $k = 3h + w/2$  to Wheeler’s microstrip inductance; excellent agreement is obtained over a wide range of microstrip aspect ratio.

### III. Results

The complex propagation constants of two microstrip line structures have been calculated using both full-wave and the new quasi-static model. The spectral domain approach is used for the full-wave calculations [8]. The full-wave results shown here are essentially identical to the results of Mesa *et al.* [9].

The first example is the case considered originally by Hasegawa [3], which has frequently been used by others as a standard for comparison. The structure consists of a 160  $\mu\text{m}$  wide microstrip, on a 1  $\mu\text{m}$  thick silicon dioxide layer, on a 250  $\mu\text{m}$  thick silicon substrate. Using the quasi-static model discussed above, the surfaces of attenuation constant,  $\alpha$ , and slow wave factor,  $\beta/\beta_0$ , as a function of both  $f$  and  $\sigma$  are shown in Figs. 3 and 4. Also shown in each figure are specific contours found using the full-wave calculations. For the attenuation constant,  $\alpha$  (Fig. 3), the agreement between the quasi-static and full-wave calculations is excellent over the full four orders of magnitude of frequency and conductivity shown, covering all three domains of skin-effect, slow wave, and dielectric quasi-TEM propagation. For the slow wave factor,  $\beta/\beta_0$ , (Fig. 4) only at the very highest frequency and conductivity is there a noticeable difference (which is still less than 20 %). In contrast, Mesa *et al.* [9], who used a more conventional quasi-static model which did not fully consider the impact of the semiconductor on  $Z$ , showed significant disagreement between full-wave and their quasi-static results, even at low frequency and conductivity. Since the quasi-static models used in [9] were not correct, there is in fact no support for their argument that quasi-static models fail to correctly predict the behavior of these transmission lines. Our improved quasi-static model produces excellent agreement to full wave calculations, and has been achieved simply by assuring that the quasi-static model is physically correct, i.e., it accounts for changes in both the shunt admittance per unit length  $Y$  **and** the series impedance per unit length  $Z$ , including the dimensional non-uniformity in cross section (the “field spreading”).

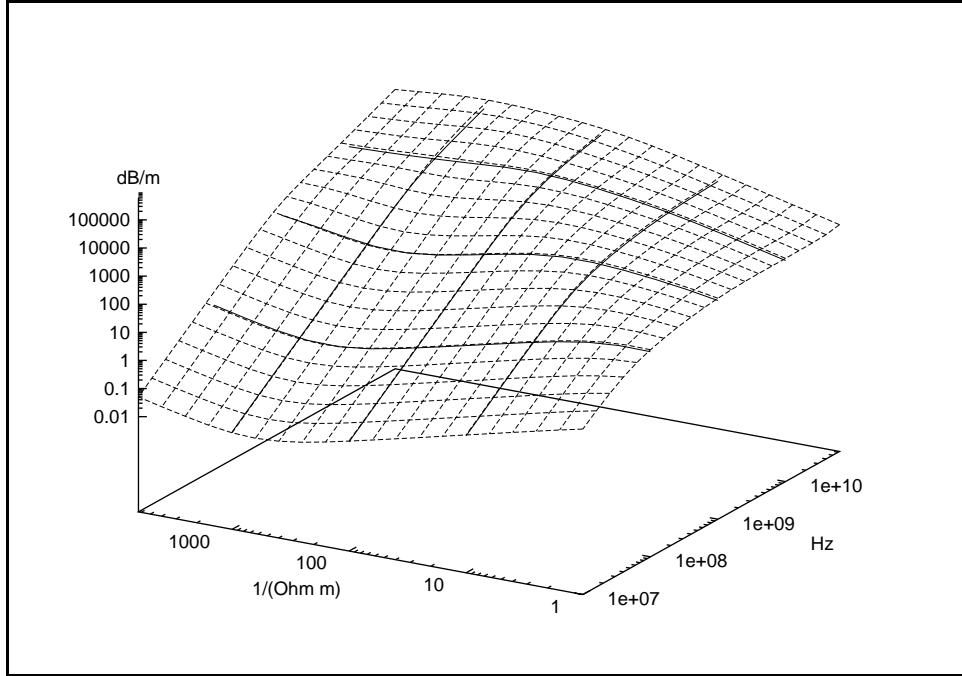


Figure 3: Surface of attenuation constant  $\alpha$  versus conductivity and frequency for microstrip geometry in [3], dotted lines: new quasi-static model for  $k = 3h + w/2$ ; solid lines: full-wave results.

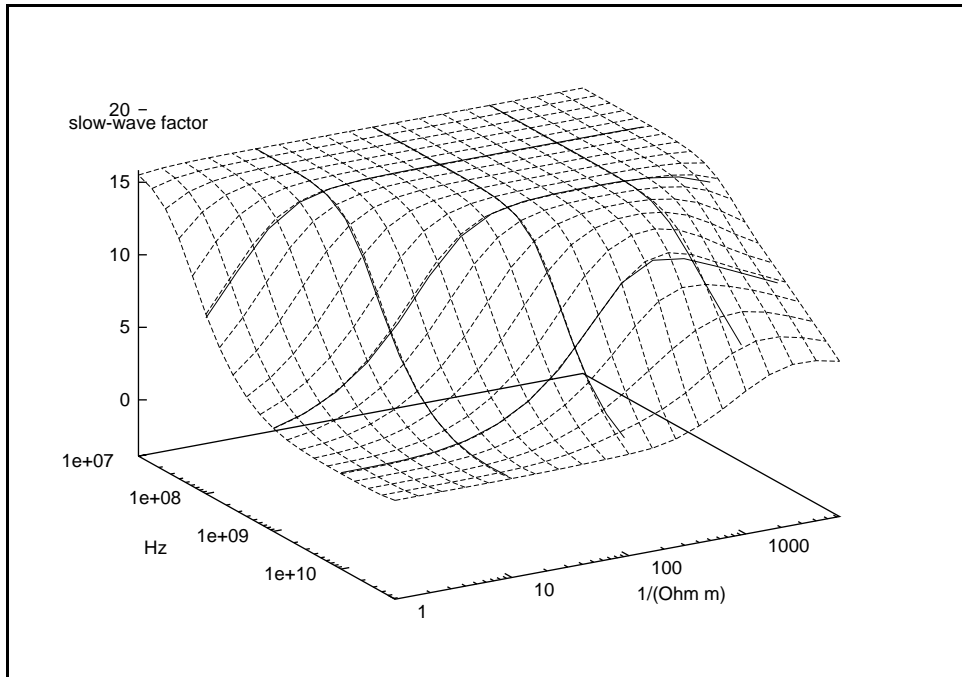


Figure 4: Surface of slow wave factor  $\beta/\beta_0$  (i.e., the phase constant normalized to the phase constant of free space) versus conductivity and frequency for microstrip geometry in [3]; dotted lines: new quasi-static model for  $k = 3h + w/2$ ; solid lines: full-wave results.

We have also verified the  $h/w$  and conductivity dependence of the model, keeping the frequency fixed at 1 GHz. For this example the linewidth is held constant at 50  $\mu\text{m}$ , the thickness of the silicon dioxide layer is 1  $\mu\text{m}$ , and the thickness of the silicon layer is varied from 10  $\mu\text{m}$  to 1000  $\mu\text{m}$ . For both the attenuation constant  $\alpha$  and phase constant  $\beta$  (phase constant results shown below in Fig. 5), the quasi-static calculation was typically within 5 % of the full-wave result, for a range of conductivity from 0.01 to 10 S/cm. Again, the agreement between the full wave and quasi-static calculations is due to the use of eqs. 4 and 5 to find the surface impedance of the lossy semiconductor layer.

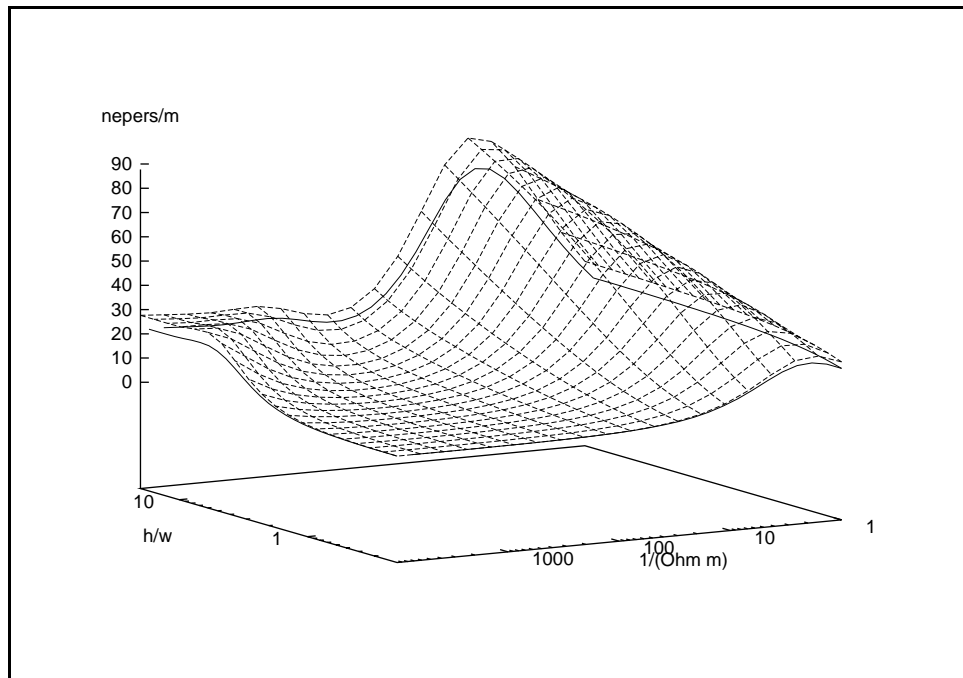


Figure 5: Surface of slow wave factor  $\beta/\beta_0$  (i.e., the phase constant normalized to the phase constant of free space) versus conductivity and microstrip geometry at a fixed frequency of 1 GHz; dotted lines: new quasi-static model for  $k = 3h + w/2$ ; solid lines: full-wave results.

#### IV. Conclusions

An accurate quasi-static model of a microstrip over a semiconductor layer has been developed. The model agrees with full wave calculations in all three modes of propagation (skin-effect, slow wave, and dielectric quasi-TEM), for both the attenuation constant  $\alpha$  and the propagation constant  $\beta$ . The agreement between quasi-static and full wave models suggests that

even for the high frequency, high conductivity case, the behavior of the transmission line is still approximately quasi-TEM. The closed form equations used here lead to extremely rapid evaluation of MIS-microstrip transmission line propagation constants.

### Acknowledgments

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### References

- [1] H. Guckel, P. A. Brennan, and I. Palocz, "A parallel plate waveguide approach to microminiaturized, planar transmission lines for integrated-circuit chips," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-15, pp. 468-476, 1967.
- [2] I. T. Ho and S. K. Mullick, "Analysis of transmission lines on integrated-circuit chips," *IEEE Journal of Solid-State Circuits*, vol. SC-2, pp. 201-208, 1967.
- [3] H. Hasegawa, M. Furukawa, and H. Yanai, "Properties of microstrip line on Si-SiO<sub>2</sub> system," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-19, pp. 869-881, 1971.
- [4] L. D. Landau and E. M. Lifshitz, in *Electrodynamics of continuous media*, vol. 8, *Course of Theoretical Physics*: PERGAMON PRESS, 1960, pp. 256-262.
- [5] H. Hasegawa and S. Seki, "Analysis of interconnection delay on very high-speed LSI/VLSI chips using an MIS microstrip line model," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-32, pp. 1721-1727, 1984.
- [6] R. A. Lawton and W. T. Anderson, "Two-layer dielectric microstrip line structure: SiO<sub>2</sub> on Si and GaAs on Si: modeling and measurement," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-36, pp. 785-789, 1988.
- [7] H. Wheeler, "Transmission-Line Properties of a Strip on a Dielectric Sheet on a Plane," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-25, pp. 631-647, 1977.
- [8] T. Itoh, "Numerical techniques for microwave and millimeter-wave passive structures," : JOHN WILEY & SONS, 1989.
- [9] F. L. Mesa, G. Cano, F. Medina, R. Marques, and M. Horno, "On the quasi-TEM and full-wave approaches applied to coplanar multistrip on lossy dielectric layered media," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-40, pp. 524-531, 1992.