

SIMIAN USER'S GUIDE

(version 2.1)

Surface Impedance Method for Interconnect Analysis

(so simple, even a monkey can do it)

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1 Introduction

SIMIAN (Surface Impedance Method for Interconnect AnalysIs) is a two dimensional frequency dependent series impedance extraction tool for interconnects and transmission lines using conductors of rectangular or circular cross section. Unlike the Volume Filament Method [1], only the surface of the conductor is divided, which can save significant amounts of computation time, especially when the cross sectional size of the conductor(s) is (are) comparable to the skin depth [2]. This method has also been extended to solve three dimensional problems [3]. SIMIAN returns the frequency dependent series impedance matrix for an n-conductor system, i.e., the $n \times n$ matrix consisting of self and mutual resistances and self and mutual inductances of and between all n conductors. You now also have the option to specify "loop impedance" as the desired output, which is more like the "per unit length" impedance usually used in transmission line analysis (see section 7.1).

1.1 Some new features in version 2.1.

1.1.1 Some bugs have been fixed... (the never ending quest...).

1.1.2 Circular conductors are now supported.

1.1.3 Loop impedance (as opposed to partial impedance) can be directly calculated.

1.1.4 The volume filament technique (VFM) can now be used as an option.

See the section "Effective Internal Impedance (EII) 'type'" on page 8 for details.

2 Input file format

There are three separate input parts: Comment, global variables specification, and conductor specification. The following is an input file that calculates series impedance of two conductors with cross sectional geometry shown in Figure 1.

Input File:

```
* Example 1
* 20μm x 20μm conductor separated by 5 μm
* This is a comment
* CASE INSENSITIVE
* Following is frequency specification
.freq 1e5 1e11 5

* Following is unit specification
.unit m

* Following is output format option
.partial yes

*Following is first conductor specification
Line {
v=0
x1=0 y1=0
x2=20e-6 y2=20e-6
rx= 1.2 ry=1.2
sigma=5.8e7
type=PW
}

* Following is second line specification
Line {
v=1
x1=25e-6 y1=0
x2=45e-6 y2=20e-6
sigma=5.8e7
}
end
```

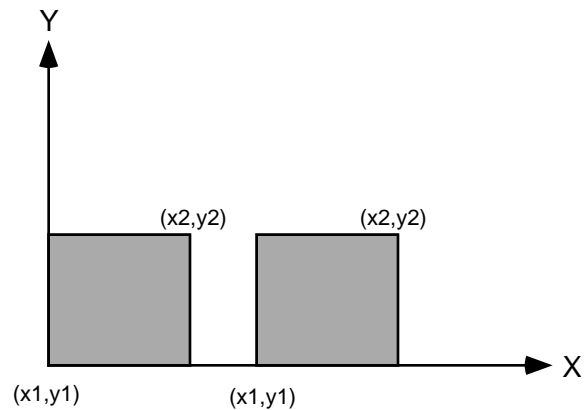


Figure 1: Example 1, Two conductor case.

2.1 Comments

Comments always preceded by “*” on each comment line. Comments can be made anywhere except inside conductor specification (inside the brackets: {}).

2.2 Global Variables

There are three global variables in SIMIAN; these variables are preceded by a period: '.' One global variable determines the frequency range over which the calculation is performed. The syntax is:

.Freq <fmin> <fmax> <points>

<fmin> is the minimum frequency, <fmax> is the maximum frequency, and <points> determines the number of frequency points between <fmin> and <fmax>. The steps between frequency points are determined logarithmically, and the impedance matrix will be calculated at a total of **points+1** sample frequencies including **fmin** and **fmax**.

The second global variable specifies the unit of length used: **m** (meter), **mm** (millimeter), **cm** (centimeter), **um** (micrometer), **in** (inch), and **mils** (0.001 inch). Since this version of SIMIAN is only for two dimensions, all the series impedance results are per unit length. This variable determines the units used in input file and output files. The default is **m** (meter), and the syntax is

.Unit <unit>

The unit of conductivity is siemens per <unit>.

The third global variable is related to output format: partial impedance matrix or loop impedance matrix. The partial series impedance is the impedance of a line assuming current returns "somewhere at infinity" and is most likely useful for inductor design. For the loop series impedance calculation, the user needs to specify at least one "ground line" by assigning zero voltage (please see Conductor Variables below). The dimension of the matrix for partial impedance is $N \times N$, where N is total number of conductors, while for the loop impedance matrix it is $N_s \times N_s$, where N_s is the total number of signal conductors (all the conductors with nonzero voltage assigned are considered as signal conductors, see 2.3 below). This option can be used for transmission lines and interconnects. The syntax is

.partial <yes/no>

The default is '.partial no'

2.3 Conductor Variables

Two different conductor cross-sections are supported by SIMIAN: rectangular and circular. Each conductor in the n-conductor system has a statement describing its characteristics in the following form:

For rectangular cross-section:

```
* rectangular line specification
Line {
v=0
x1=0 y1=0
```

```

x2=20e-6 y2=20e-6
rx= 1.2 ry=1.2
sigma=5.8e7
type=PW
}

```

For circular cross-section:

```

* circular shape conductor
* xc and yc is coordinate of the center and rad is radius of the circle
circle {
v=1
xc=0 yc=0
rad=20e-6
nr=10
sigma=3e7
}

```

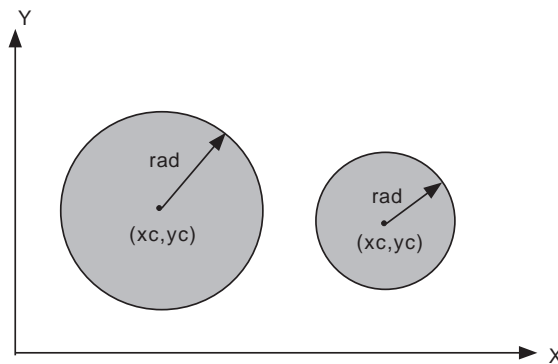


Figure 2: Circular cross-section specifications.

Conductor shape: The first variable to specify is the conductor shape. There are three conductor shapes in SIMIAN. For rectangular shape conductor one can use either ‘**Line**’ or ‘**Plate**’ option. This variable must be specified just before the bracket ‘{’. ‘**Line**’ can be used in most rectangular or square shaped conductor cases when the conductor shape is not a “wide strip” (like a ground plane), or high accuracy is required. ‘**Plate**’ can be used for wide strips or ground planes (i.e., when the conductor is much wider than it is thick); for a ‘**plate**’ SIMIAN automatically determines the segmentation (i.e., ribbon width) so that computation time can be reduced [4]. An example using this type will be explained in section "Segmentation Scheme"; this option can be efficiently used especially when *width/height* ratio is bigger than 1, where *height* is distance between signal line and ground plane and *width* is the width of the signal conductor.

For a circular conductor, the user should use ‘**circle**’. SIMIAN approximates the circular-shaped conductor using a polygon with ‘nr’ vertices (see Number of Segments below).

All the rest of the individual conductor characteristics are specified between two brackets: '{' and '}' :

Voltage applied: Voltage applied on each line. SIMIAN distinguishes only zero voltage from non-zero voltage. This voltage option is used in conjunction with '.partial <no/yes>' option described in section 2.2. Zero voltage assigned conductors are considered as grounds, and all non-zero voltage assigned conductors are considered as signal lines.

v=<voltage in V>

Geometry Input: As shown in Figure 1, the rectangular geometry of the each conductor can be represented by two sets of coordinates: the lower left corner at (**x1,y1**) and upper right corner at (**x2,y2**) of the rectangular cross-section conductor. Since this version of SIMIAN is two dimensional, z coordinates are not necessary. Figure 2 shows circular conductors. For circular conductors the user needs to specify coordinates of the center of each conductor, **xc** and **yc**, and the radius of each conductor, **rad**.

Non-uniform Segmentation: This option is only available with 'Line' option. Conductors with high aspect ratio (width/thickness) have highly non-uniform current distribution at high frequency. To capture these non-uniformities, the conductor has to be segmented into finer sections to capture these effects (especially at high frequency). To avoid having too many ribbons, the conductor can be segmented non-uniformly (see Figure 2) using finer segmentation near corners. The syntax is

rx= <ratio in x direction> ry=<ratio in y direction>

'**rx**' and '**ry**' are the ratios of two adjacent ribbon lengths in the x and y directions, respectively (ribbons near corners are always shortest). Defaults are rx=1 and ry=1 (uniform segmentation). In Figure 3, rx=2 and ry=1 (uniform segmentation) is used, with 5 segments in the x direction, 2 segments used in the y direction.



Figure 3: Nonuniform segmentation using rx=2 and ry=1 with nx=5 and ny=2

Number of Segments: The number of ribbons used on each conductor face influences both accuracy and computation time. These variables for rectangular conductors are specified as

nx= <# of segments in x direction>

ny= <# of segments in y direction>

and for the circular conductor as

nr=<# of segments used along the perimeter>

We have found that with our approach it is rarely necessary to use a large number of ribbons, even at very high frequencies. Default values are nx=5, ny=5 for rectangular conductor and nr=10 for circular conductor, which we have found to give good accuracy over wide ranges of frequency and geometry. In many case reasonable accuracy can be achieved with as few as one ribbon per face, i.e., nx=1, ny=1; this is particularly useful for an n-conductor system when n is large but fast computation is still desired. **For a circular conductor, at least 4 ribbons are necessary.**

Effective Internal Impedance (EII) 'type': SIMIAN uses an impedance boundary condition [2] to represent the interior of each conductor, called the effective internal impedance (EII). This position dependent value is assigned to the thin ribbons on the surface of the conductors. There are several ways to divide the conductor to get an approximation to the EII [2]; this program uses the "transmission line model" and "modified plane wave model." The syntax is

type=<EII type>

where the EII type can be 'TL' (transmission line model), 'PW' (plane wave model), or 'VF' (volume filament technique [1, 2]). The plane wave model is somewhat more numerically stable, while the transmission line model is more accurate; both are approximations of the effective internal impedance (EII) used in the surface ribbon method (SRM). For a more "rigorous" approach (for people who don't trust SRM or who want to compare the results between the SRM and the volume filament method (VFM)), the user can use the conventional volume filament technique [1, 2] instead by using the 'VF' option. The 'number of segments specification' using 'nx' and 'ny' now becomes the number of filaments used in x and y directions.

****Caution:** Unlike SRM, VFM may need many filaments; the number of filaments required is usually determined by frequency of interest and the conductor dimensions relative to the skin depth. For instance, the following restriction on the number of filaments (and hence on filament size) is fairly safe:

$$nx \geq |x_2 - x_1| \cdot \sqrt{\pi f \mu \sigma}$$

$$ny \geq |y_2 - y_1| \cdot \sqrt{\pi f \mu \sigma} .$$

In some cases, the user may want to use both SRM and VFM together. For instance, in a multi-conductor system, where some conductor dimensions are small enough that skin effect is not important, one can use the 'VF' option with $nx = ny = 1$ (Simian automatically uses the 'VF' option in this case), while SRM with 'TL' or 'PW' options can be used for the conductors whose dimensions are larger than or comparable to the skin depth.

The default option is 'TL'.

'VF' option is not supported for circular conductor.

Sigma: conductivity of the conductor. Syntax is

sigma= <sigma>

End: All the files should be terminated with 'end'

3 Segmentation Scheme

Since the total number of ribbons used for each conductor determines the computation time, it is very important to reduce them without loss of accuracy. Since SIMIAN only divides the conductor on its surface, the problem can be reduced from N^2 to $4N$ compared to the volume filament method. However, more significant advantage comes from reduction of N itself. To get accurate result in the volume filament method, each segment has to be comparable in size to the skin depth at the frequency of interest; if a filament is much larger than about 1/3 of a skin depth, serious error can result. However, in the surface ribbon method a much smaller number of segments can be used while maintaining reasonable accuracy (typically less than 1% error at "low" frequencies, perhaps as much as 10% over a narrow band of frequencies at "mid" frequency, and again about 1% at "high" frequency). For most cases using more than five ribbons on each side would not be necessary, and in many cases as few as one per face is necessary.

Figure 4 shows minimum segmentation method using 'plate' option in Conductor type variable applied to ground plane. Only one ribbon is used on each side of signal lines, and eight for the "ground plane" (segmented automatically using the 'plate' option), resulting in a total of 20 ribbons. The input file for this example is shown below.

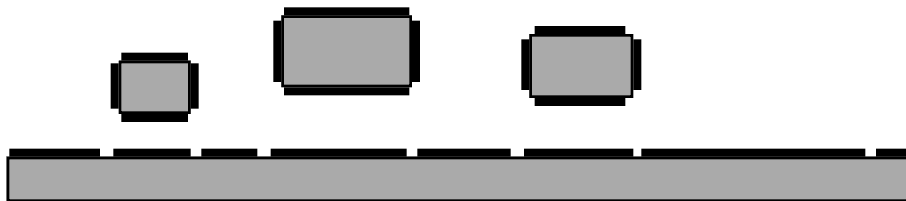


Figure 4: Minimum Segmentation Scheme with ground plane.

```
* Example 2
* 3 conductor lines and one ground plane
.freq 1e5 1e11 20
.unit m
.partial no

line {
v=1
```

```

x1=1.5e-5 y1=1.8e-5
x2=2.7e-5 y2=2.8e-5
sigma=5.8e7
nx=1 ny=1
}

line {
v=1
x1= 3.7e-5 x2=5.7e-5
y1= 2.6e-5 y2=3.8e-5
sigma=5.8e7
nx=1 ny=1
}

line {
v=1
x1=7.2e-5 x2=8.8e-5
y1=2.4e-5 y2= 3.4e-5
sigma=5.8e7
nx=1 ny=1
}

plate{
v=0
x1= 0 x2= 13e-5
y1= 0 y2=1e-5
sigma=5.8e7
}

end

```

4 Output File

The usage of SIMIAN is

simian <Input file name> <w/r>

‘w’ option indicates there has been no previous calculation on the same geometry; the program creates ‘**induc**’ for future use. ‘r’ option uses old ‘**induc**’ to redo the calculation over a new frequency band for the previously run geometry. Since the inductances between ribbons are geometry dependent, **but not frequency dependent**, this reduces over all computation time.

There are three output files produced by SIMIAN.

1. **induc**: produces self and mutual inductances between ribbons. The purpose of this file is to reuse these values when testing the same geometry over different frequency ranges.
2. **plate**: When the user decides to use the ‘Plate’ option to extract series impedance of a ground plane, the ribbon segmentation scheme automatically determines positions of ribbons on ground plane. This file contains that information.
3. **Zs**: Impedance matrix (Resistance and Inductance).

The following is the Impedance matrix result ('Zs') for Example 1 using SIMIAN:

Impedance 2 x 2 matrix (R,L) at f=1.0000e+05 Hz
(4.3103e+01, 2.3235e-06) (-2.2987e-07, 2.1178e-06)
(-2.2987e-07, 2.1178e-06) (4.3103e+01, 2.3235e-06)

Impedance 2 x 2 matrix (R,L) at f=1.5849e+06 Hz
(4.3116e+01, 2.3234e-06) (-6.0394e-05, 2.1178e-06)
(-6.0394e-05, 2.1178e-06) (4.3116e+01, 2.3235e-06)

Impedance 2 x 2 matrix (R,L) at f=2.5119e+07 Hz
(4.5583e+01, 2.3168e-06) (-1.1122e-01, 2.1174e-06)
(-1.1122e-01, 2.1174e-06) (4.5583e+01, 2.3169e-06)

Impedance 2 x 2 matrix (R,L) at f=3.9811e+08 Hz
(1.0982e+02, 2.2674e-06) (-6.7482e+00, 2.1225e-06)
(-6.7482e+00, 2.1225e-06) (1.1022e+02, 2.2672e-06)

Impedance 2 x 2 matrix (R,L) at f=6.3096e+09 Hz
(4.1038e+02, 2.2348e-06) (-5.1520e+01, 2.1261e-06)
(-5.1520e+01, 2.1261e-06) (4.0350e+02, 2.2350e-06)

Impedance 2 x 2 matrix (R,L) at f=1.0000e+11 Hz
(1.7044e+03, 2.2275e-06) (-2.5654e+02, 2.1273e-06)
(-2.5654e+02, 2.1273e-06) (1.7000e+03, 2.2276e-06)

5 Examples and Results

5.1 Square Twin Lead

Example 1 is simulated with SIMIAN and compared with FastHenry [5] which uses the volume filament method accelerated using a multi-pole algorithm. As Figure 5 shows, SIMIAN can reduce matrix size significantly. For the problem given, SIMIAN only uses 30 to 40 total ribbons while the volume filament method needs more than 800 filaments to accurately predict total resistance at high frequency. FastHenry accelerates matrix calculation with the help of a multi-pole algorithm, and this same algorithm can also be applied to the surface ribbon method. Figure 6 shows the results of FastHenry, SIMIAN, and the surface ribbon method (SRM) using a multi-pole algorithm. A total 40 ribbons were used for SIMIAN and SRM with multi-pole algorithm, while 800 filaments were used for FastHenry. In this particular example, the time advantage can be 400 to 8000 times faster than using the volume filament method, depending on what method is used for matrix calculation. Considering this is for only one frequency point, tremendous time savings can be achieved by using the SRM.

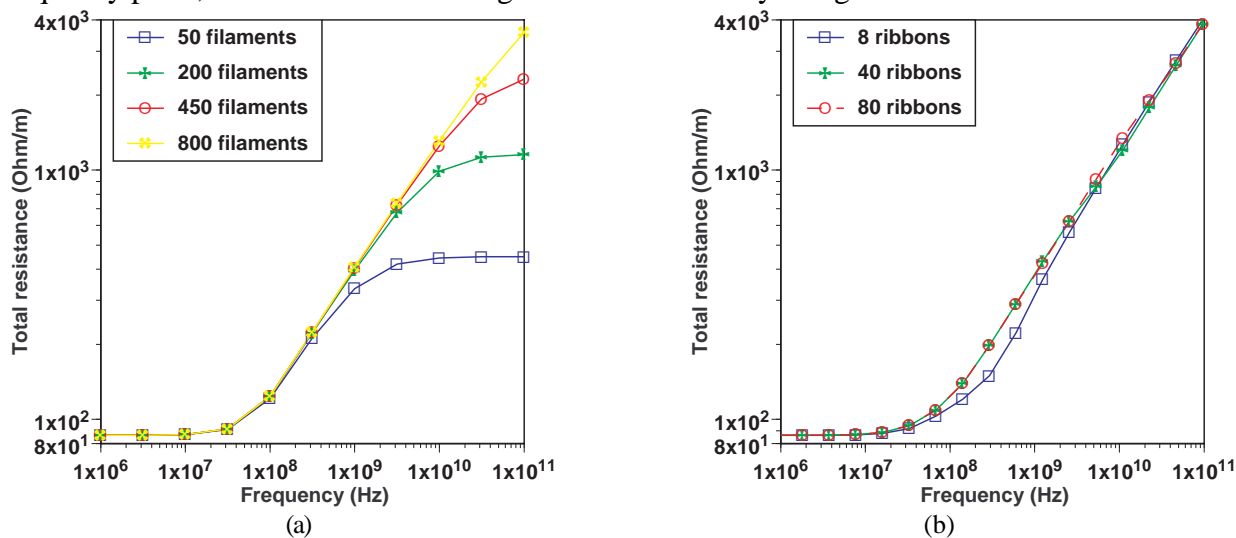


Figure 5: (a) FastHenry [5] results versus number of filaments; (b) SIMIAN versus number of ribbons.

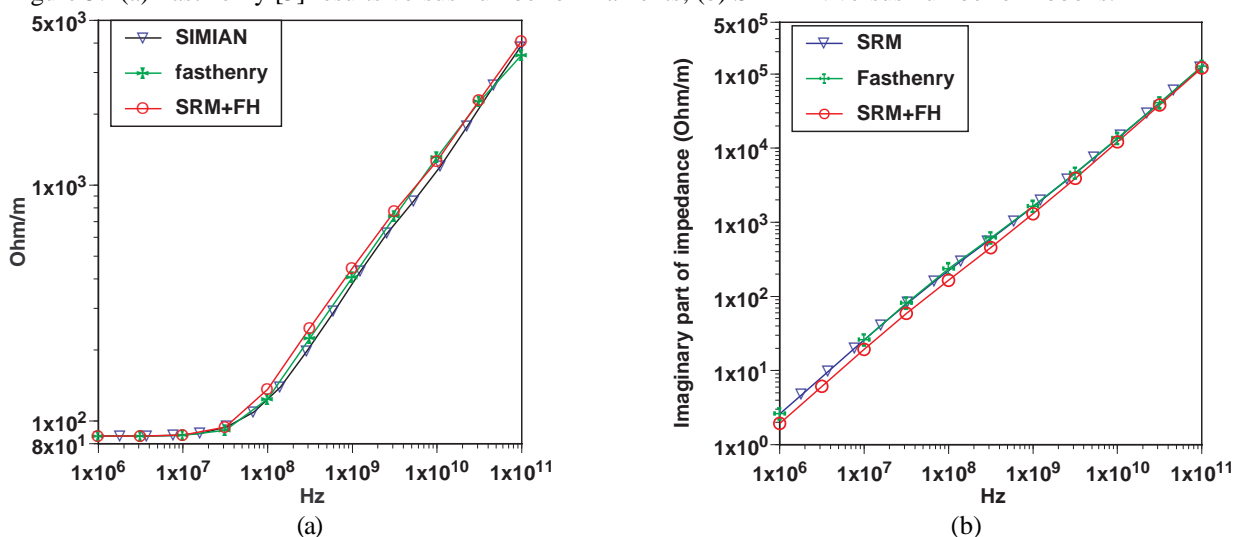
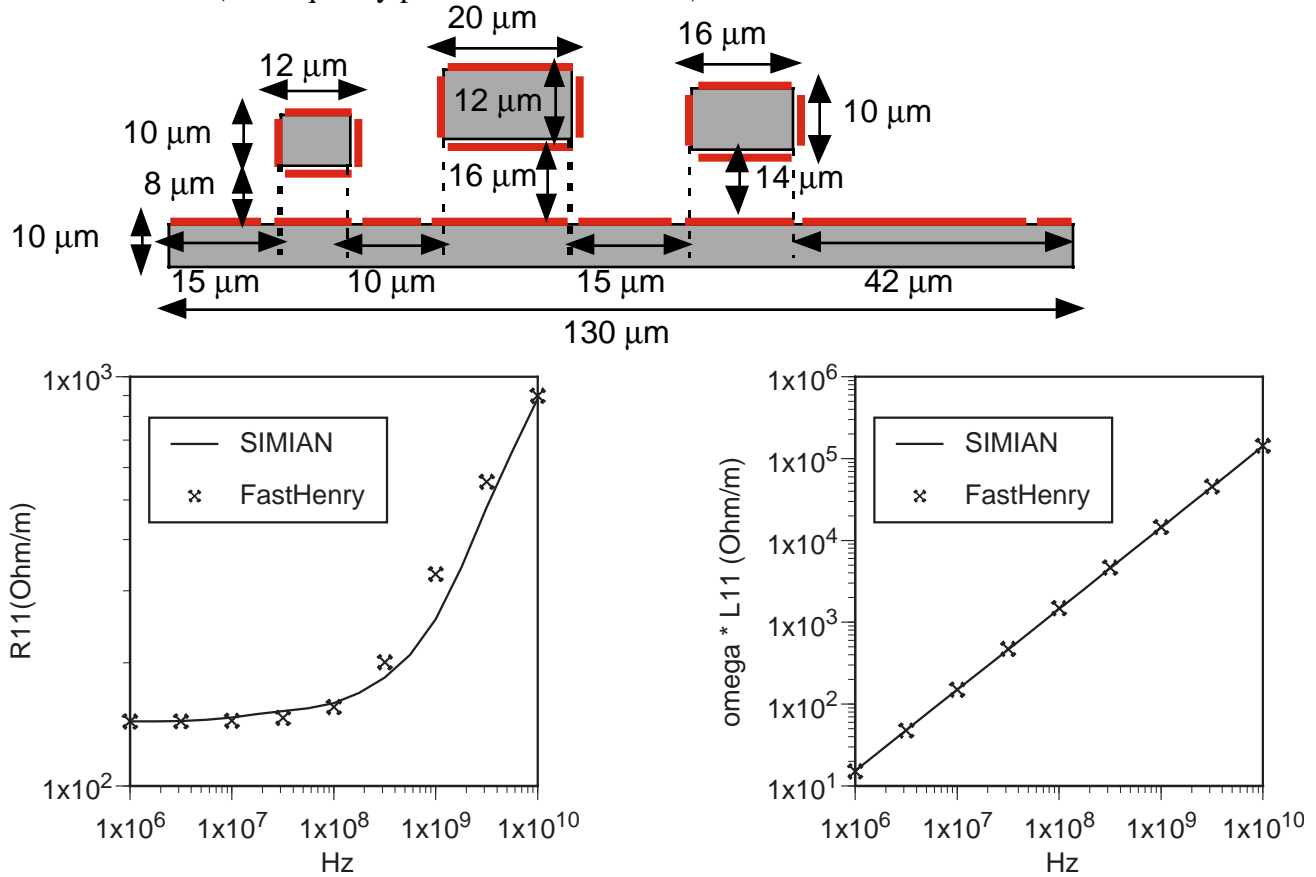


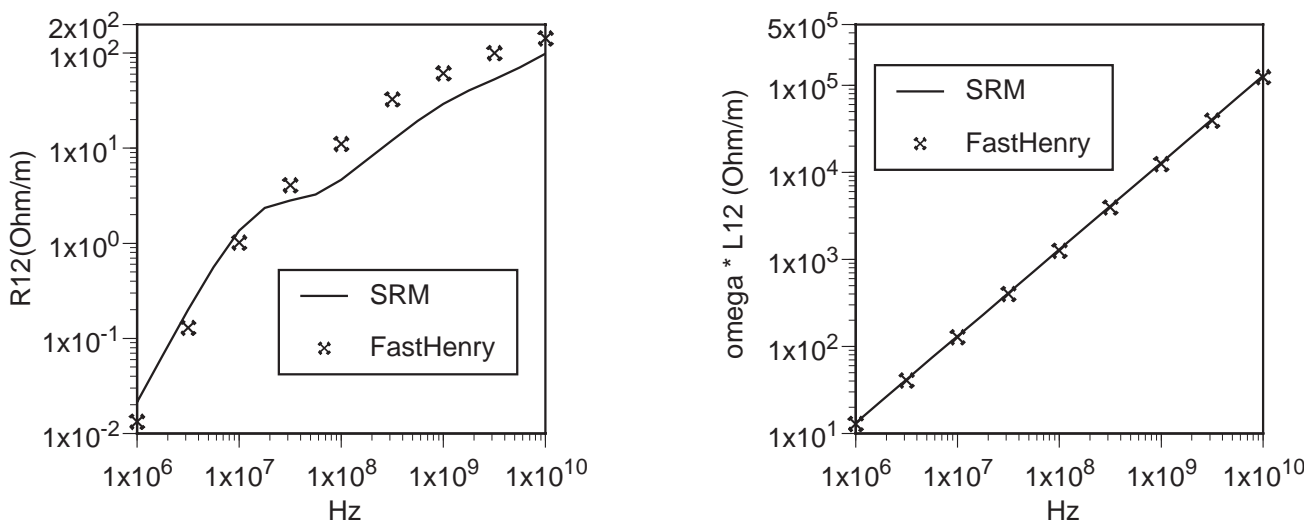
Figure 6: solid line: SIMIAN (40 ribbons); circle: SRM with multi-pole algorithm (40 ribbons); cross: FastHenry (800 filaments). (a) Real part of total Impedance (R). (b) Imaginary part of total Impedance (ωL).

5.2 Three Conductors over Finite Size/Conductivity Ground Plane

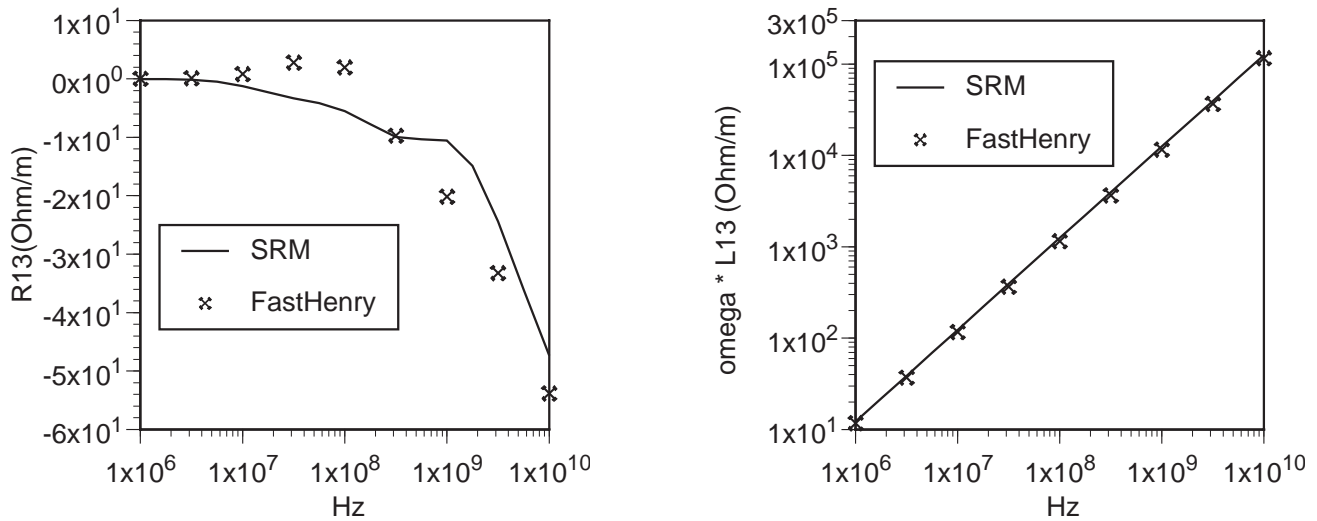
Figure 7 shows results from simulating Example 2. The total number of ribbons used in SIMIAN was 2 which was considerably less than FastHenry (1250). Reasonable accuracy was achieved through SIMIA within a second (20 frequency points with SPARC 20).



(a) Example 2, Z11



(b) Example 2, Z12



(c) Example 2, Z13

Figure 7: Real and Imaginary part of Series Impedance of Example 2 (three conductors over a ground plane).

6 Notes

This SIMIAN program was successfully compiled with ANSI C(g++). The platforms tested thus far include:

1. Sparc 20 machine with Sunos 4.1.3
2. Sparc 20 machine with Solaris 2.5
3. Pentium machine with Linux
4. HP workstation

To compile, just type '**make**' after you untar the file. The authors of SIMIAN are not responsible for successful compilation, so good luck on your machine! We also distribute binary files of SIMIAN.

7 Appendices

7.1 Appendix 1: Conversion between Z-matrix form and conventional transmission line series impedance per unit length.

7.1.1 Two conductor case

Let A and B represent planes in which measurements of voltage or current can be made. Consider A to be the “input plane” and B to be the “output plane”.

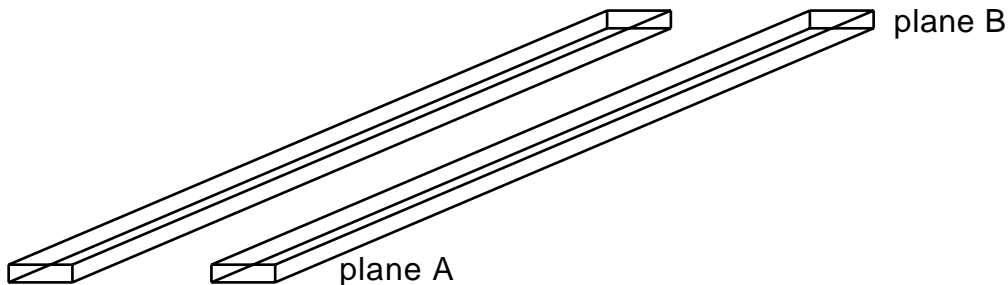


Figure 8: Two conductor example for conversion to transmission line formulation.

The impedance matrix formalism for such a two conductor system is written in terms of the voltages across each conductor between planes A and B, and in terms of the current flowing down each conductor (shown in Figure 9). Note in conventional transmission lines the “return path” is normally shown as an ideal “ground,” and voltages should ONLY be specified within a single plane; voltage differences between two different planes should not be considered (see Figure 10).

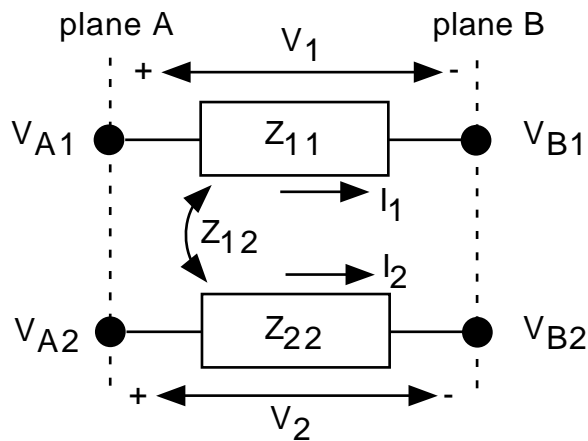


Figure 9: Z matrix representation for two conductors beginning in plane A and ending in plane B.

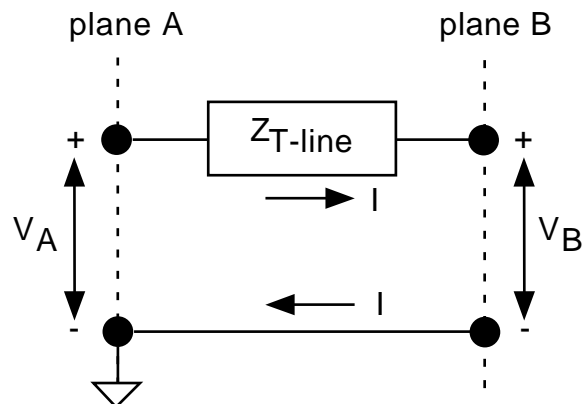


Figure 10: T-line series impedance only.

In the impedance matrix formalism the results are summarized by:

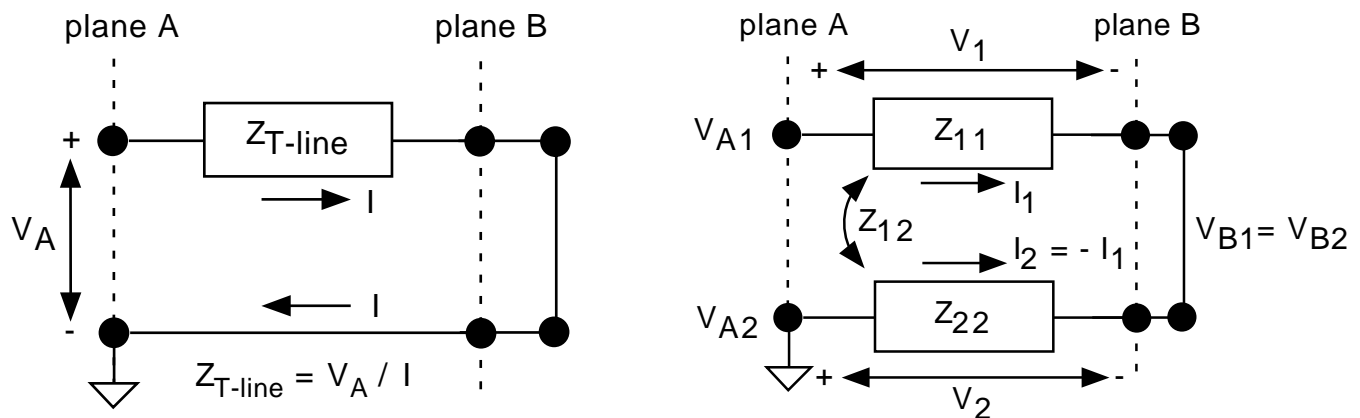
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Note that this problem is assumed to be reciprocal (as it would be for any “normal” situation), so that $Z_{21} = Z_{12}$.

In the T-line formalism, since the return path is assumed to be ideal (i.e., no voltage drops occur in the “ground line”):

$$Z_{T-line} \cdot I = V_A - V_B$$

To convert from the impedance matrix formulation to the conventional T-line formulation we need to find a relation between V_1, V_2, I_1, I_2 in the Z-matrix form and $V_A, V_B,$ and I in the T-line form. To do this consider the case where a “short” is placed between the conductors in plane B and conductor 2 is grounded in plane A, as shown below:



We now have for the T-line:

$$V_B = 0 \quad \text{that then yields: } Z_{T-line} = \frac{V_A}{I}$$

and for the Z-matrix:

$$V_{B2} = V_{B1}; \quad V_{A2} = 0; \quad I_2 = -I_1, \quad \text{that then yields: } V_1 = V_{A1} - V_{B1} \quad .$$

The connection between the two formalisms is:

$$I_1 = I \quad \text{and} \quad V_A = V_{A1}, \quad \text{or finally } Z_{T-line} = \frac{V_{A1}}{I_1} \quad .$$

We can now evaluate the impedance matrix product:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ -I_1 \end{bmatrix} = \begin{bmatrix} V_{A1} - V_{B1} \\ -V_{B1} \end{bmatrix}$$

to eliminate V_{B1} . Solving for V_{A1} in terms of I_1 finally gives:

$$Z_{T-line} = Z_{11} + Z_{22} - 2 \cdot Z_{12} \quad .$$

7.1.2 Three conductor case

Consider a three conductor system used as a transmission line, with two conductors (labeled 2 and 3) connected together as ground. The equivalent Z-matrix circuit is shown below:

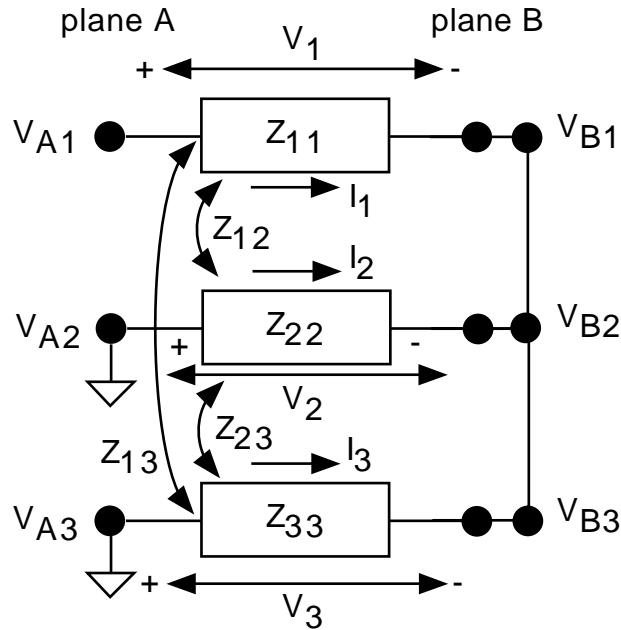


Figure 11: Three conductor Z to T-line conversion.

This gives:

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{22} & Z_{23} \\ Z_{13} & Z_{23} & Z_{33} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ -(I_1 + I_2) \end{bmatrix} = \begin{bmatrix} V_{A1} - V_{B1} \\ -V_{B1} \\ -V_{B1} \end{bmatrix}$$

Solving for the current in conductor 2 gives:

$$I_2 = I_1 \cdot \frac{Z_{33} + Z_{12} - Z_{13} - Z_{23}}{2 \cdot Z_{23} - Z_{22} - Z_{33}}$$

And finally solving for V_{A1} yields:

$$Z_{T-line} = \frac{V_{A1}}{I_1} = Z_{11} + Z_{23} - Z_{12} - Z_{13} + \frac{(Z_{33} + Z_{12} - Z_{13} - Z_{23}) \cdot (Z_{23} + Z_{12} - Z_{13} - Z_{22})}{2 \cdot Z_{23} - Z_{22} - Z_{33}}$$

8 References

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