Chapter 2

Transmission Lines on Lossy Substrates

For the last twenty-five years there has been a great deal of interest in modeling micro-strip transmission lines on semiconducting substrates. On-chip and some multichip module[1] interconnects, that fall in this category, produce behavior that is more difficult to predict than that of lines made on lossless substrates. Microstrip transmission lines on layered lossy substrates can support fundamentally different modes of propagation [13, 17], which lead to a "de coupling" of the inductance L and capacitance C per unit length for the microstrip. Overall, a significant change in the signal propagation velocity can result with changing geometry, medium parameters, or frequency. Typically there is a range of parameters where the capacitance of the line is increased, producing a propagation velocity much slower than expected. In 1971 Hasegawa et al. [14] experimentally verified this behavior for a microstrip on a SiO_2-Si substrate, over a wide range of substrate resistivity, dielectric thickness, and microstrip width. Quasi-statics accurately predicts the behavior of such transmission lines, with excellent agreement between full-wave and static model over a very wide ranges of dimensions, substrate conductivity, and frequency. A modified model

[1] Multichip modules, also called MCMs, provide convenient packaging for high speed digital interconnects.

is developed using the transverse resonance method for series impedance, and Wheeler's formulae for shunt admittance[41].

2.1 Transmission Lines

Interconnect structures used on MCMs or PCBs can be considered as transmission lines, as the current return paths generally run close to the signal lines. Among the many forms of wave guiding structures, only the ones that satisfy the conditions listed below support Transverse Electro-Magnetic waves (TEM) (no field component in the direction of propagation):

- Has more than one conductor
- Embedded in homogeneous and isotropic medium
- Conductors have infinite conductivity

Actually, these conditions are never satisfied. For all practical purposes, a TEM approximation is valid for a variety of transmission line structures in the usable frequency range for that structure. If the electromagnetic waves have negligible field components in the direction of propagation compared to the transverse components, then this mode of propagation is called quasi-TEM. This approximation is valid up to the frequencies where quasi-static approximation is valid. Quasi-static fields and currents exist in structures where at the frequency of operation the wavelength is much larger than the dimensions of the structure.

Using circuit elements the behavior of the waves propagating in a transmission line can be obtained. A series impedance block will represent magnetic fields and a shunt admittance block will represent electric fields. Using the circuit elements instead of physical fields, voltages and currents can be used to characterize the transmission line. In the frequency domain voltages and currents on a transmission line operating in the quasi-TEM mode satisfy the following equations,

$$\begin{aligned} \frac{\partial V}{\partial x} &= -ZI \\ \frac{\partial I}{\partial x} &= -YV, \end{aligned} \tag{2.1}$$

where V and I represent the voltage and current on the line respectively. Z any Y are the series impedance and shunt admittance per unit lengths respectively. Solutions to Laplace's equation lead to the values for Z and Y.

Differentiating once and substituting the equations above, the wave equations for voltages and currents on the transmission line can be written as,

$$\begin{aligned} V'' - \frac{Z'}{Z}V' - ZYV &= 0 \\ I'' - \frac{Y'}{Y}I' - ZYI &= 0. \end{aligned} \tag{2.2}$$

For uniform structures series impedance Z and shunt admittance Y are constant along the propagation axis, hence the equations above simplify to uniform medium wave equations. Quasi-TEM approximation yields series impedance Z and shunt admittance Y values for a given cross-sectional geometry. Propagation parameters can be written in terms of Z and Y:

$$\begin{aligned} Z_0 &= \sqrt{\frac{Z}{Y}} \\ \gamma &= \sqrt{ZY}, \end{aligned} \tag{2.3}$$

where Z_0 and γ are the characteristic impedance and propagation constant respectively. For most cases of practical interest, the dielectric present in the

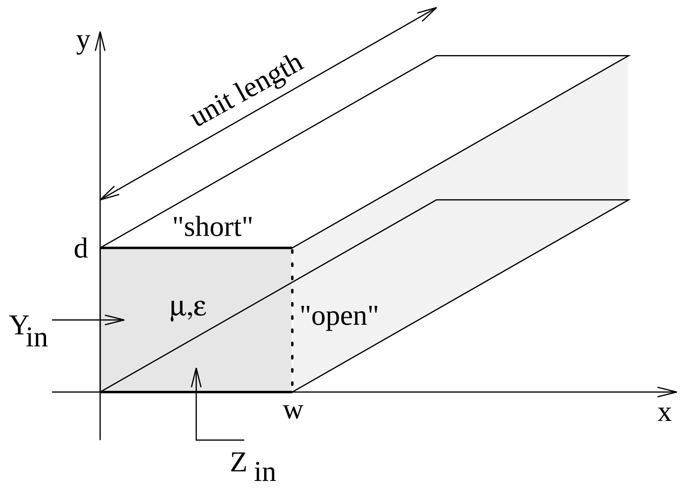

Figure 2.1: Parallel plate transmission line for the transverse resonance calculation.

surrounding space can be assumed to be lossless, hence the shunt admittance is given by:

$$Y = j\omega C \tag{2.4}$$

where C is the static capacitance per unit length of the line. Including the resistance of the conductors, series impedance is,

$$Z = R + j\omega L \tag{2.5}$$

where R and L are the resistance and inductance of the line respectively. It should be kept in mind that both R and L are in reality frequency dependent parameters.

2.2 Transverse Resonance Method

The transverse resonance method is used to calculate propagation constants of many practical structures. In this method, the cross section of the structure is represented as a transmission line system[3] in the transverse direction. The transverse resonance method has been used before to calculate transmission line characteristics including loss [6, 1]. As a simple example, let us consider a parallel plate transmission line, one plate at $y = 0$ and the other at $y = d$, of width w. Using the transverse resonance method, the inductance and capacitance of this line can be obtained. If we consider the impedance seen from the driven line, with a short representing the other line (Fig. 2.1), we have,

$$Z_{in} = \frac{1}{w}\sqrt{\frac{\mu}{\epsilon}}\tanh(j\omega\sqrt{\mu\epsilon}d) \tag{2.6}$$

where μ and ϵ are the permeability and permittivity of the filling material, respectively. Assuming transverse dimensions are much smaller than the wavelength, $\tanh(j\omega\sqrt{\mu\epsilon}d)$ can be approximated as $j\omega\sqrt{\mu\epsilon}d$. The series impedance then becomes

$$Z = j\omega\frac{\mu d}{w}. \tag{2.7}$$

We can identify $\mu d/w$ as the inductance of this parallel plate transmission line. The same approach can be used to get the shunt admittance. The admittance seen from one side of the structure looking at an open at the other side is

$$Y_{in} = \frac{1}{d}\sqrt{\frac{\epsilon}{\mu}}\tanh(j\omega\sqrt{\mu\epsilon}w). \tag{2.8}$$

At the limit the shunt admittance expression becomes

$$Y = j\omega\frac{\epsilon w}{d}, \tag{2.9}$$

also notice that, $\epsilon w/d$ is the capacitance in (2.4). For the series impedance, we can also include the effect of finite conductivity of the plates using transmission line equations for cascaded lines. This time Z_{in} is calculated with two sections (first the filling material and then the metal) of transmission line terminated with an open circuit (air boundary). This will result in frequency dependent resistance and inductance in (2.5). In the following section, use of this method is expanded to microstrip line over a multi-layer substrate.

2.3 Quasi-Static Modeling of MIS Microstrip Lines

A microstrip line on an insulating layer on a semiconductor substrate with a ground plane on the back is a Metal-Insulator Semiconductor (MIS) structure. The series impedance of the microstrip line is a function of frequency and substrate conductivity.

To determine the proper value of series impedance the effect of substrate conductivity on the magnetic field distribution must be determined. For semiconducting substrates significant magnetically induced currents occur only at very high frequencies. When the thickness of the semiconducting substrate becomes greater than the skin depth, the so-called the "skin-effect" mode of propagation is encountered [14]. Several papers in the field have recognized that this leads to a reduction in the effective separation between the signal and the ground plane, as well as inducing significant loss due to series resistance [15, 14, 27]. If the frequency or conductivity is low enough that the skin depth is larger than the thickness of the semiconductor the magnetic fields (and thus inductance L) will be determined primarily by the configuration of the metallic conductors. In the skin-effect mode, series resistance induced by the currents in the semiconducting substrate is significant.

The behavior of electric fields is quite different. If the frequency of the applied signal is below the dielectric relaxation frequency of the semiconductor, then the electric fields behave as if the semiconductor were a metallic sheet.

Therefore the capacitance C will be inversely proportional to the distance separating the metal line and the semiconductor, and nearly independent of the distance to the ground plane on the back of the substrate. Increasing the frequency or decreasing the conductivity causes the electric fields behave as if the semiconductor were a dielectric layer, leading to a reduced capacitance compared to the previous case. At the crossover region where $\omega \sim \sigma/\epsilon_{semi}$, the impact of the semiconductor conductivity on propagation loss (represented by a shunt conductance) can be very large.

2.3.1 Quasi-Static Model

The use of quasi-static models for transmission lines is well established, and has great utility when computational efficiency is required [23]. In addition, for cases where it is essential to include conductor loss, full-wave analysis can become arduous. Many quasi-static models have been proposed which adequately describe the impact of finite conductivity on shunt admittance Y, through the incorporation of a shunt conductance [25]. Equally important is the effect on the series impedance Z, in the high frequency and/or high conductivity case, which has not been properly treated from the quasi-static point of view.

The application of transverse resonance to find the equivalent series impedance of a microstrip has previously been used by [13]. However, previous work has assumed that the equivalent shunt transmission line is of uniform cross section with a short circuit boundary condition representing the perfect ground plane. This leads to the following expression for the driving point impedance,

$$Z_{eq} = \frac{j\omega\mu}{w\gamma}\tanh(\gamma h) \tag{2.10}$$

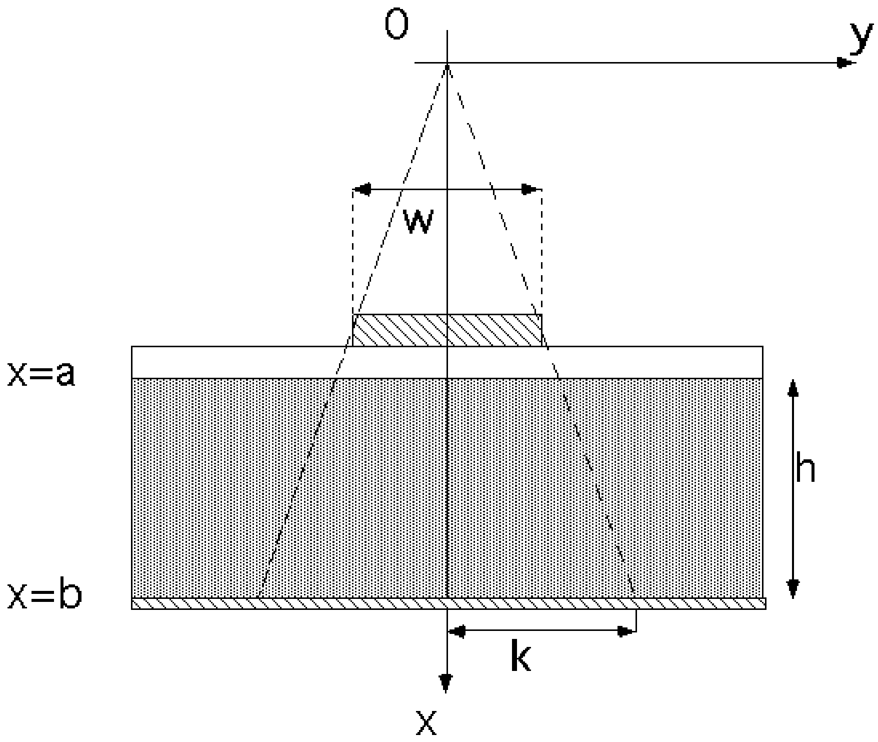

Figure 2.2: Cross section of a microstrip over a semiconducting substrate. k represents the effective spreading distance of the fields between the strip and the ground plane; best agreement between this model and conventional microstrip calculations is achieved for $k = 3h + w/2$.

where γ is the complex propagation constant in the semiconductor, h is the thickness of the semiconductor layer, w is the width of the microstrip line, and μ is the permeability of the semiconductor.

A uniform cross section approximation is invalid for microstrip except for very wide strips compared to the thickness of the substrate. A more accurate model uses a non-uniform transverse cross section, taking into account the spreading of fields. The series impedance for the microstrip is the input impedance of the short circuited transmission line shown in Fig. 2.2. If the effective cross section is assumed to vary linearly with depth x, approximating

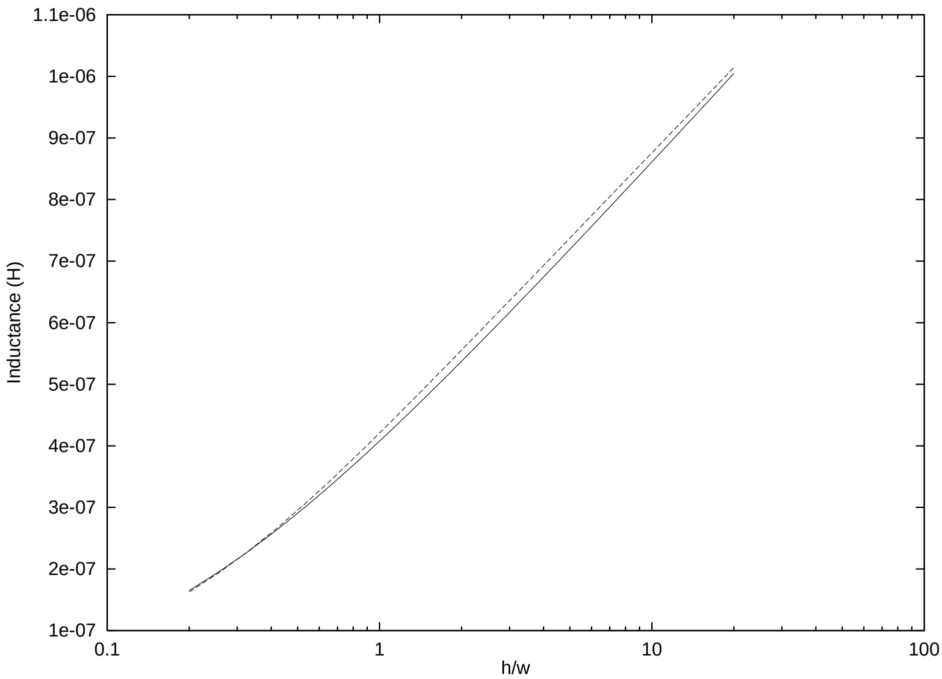

Figure 2.3: Comparison of inductance calculated using Wheeler's equation (dashed line) and (2.11) (solid line) for a range of height-width ratios.

the spreading of the fields between the microstrip and the ground plane, a closed form solution can be obtained,

$$Z_{semi} = \frac{1}{jw}\sqrt{\frac{j\omega\mu_0}{j\omega\epsilon_{semi}+\sigma}}\frac{\mathbf{H}_0^{(2)}(j\beta_s b)\mathbf{H}_0^{(1)}(j\beta_s a) - \mathbf{H}_0^{(2)}(j\beta_s a)\mathbf{H}_0^{(1)}(j\beta_s b)}{\mathbf{H}_0^{(2)}(j\beta_s b)\mathbf{H}_1^{(1)}(j\beta_s a) - \mathbf{H}_1^{(2)}(j\beta_s a)\mathbf{H}_0^{(1)}(j\beta_s b)} \tag{2.11}$$

where $\mathbf{H}_n^{(m)}$ are the Hankel functions of kind m, $\beta_s = \sqrt{j\omega\mu_0(j\omega\epsilon_{semi}+\sigma)}$, $a = (hw)/(2k-w)$, and $b = a + h$ (Fig. 2.2). The parameter k is a measure of how much the fields spread before reaching the ground plane.

In the limit of zero conductivity in the semiconductor, the impedance calculated using (2.11) should reduce to the inductance of a simple microstrip line. The inductance calculated by this method matches to that from Wheeler's equation [41] very closely (within 3%) over a wide range of h/w (height-width

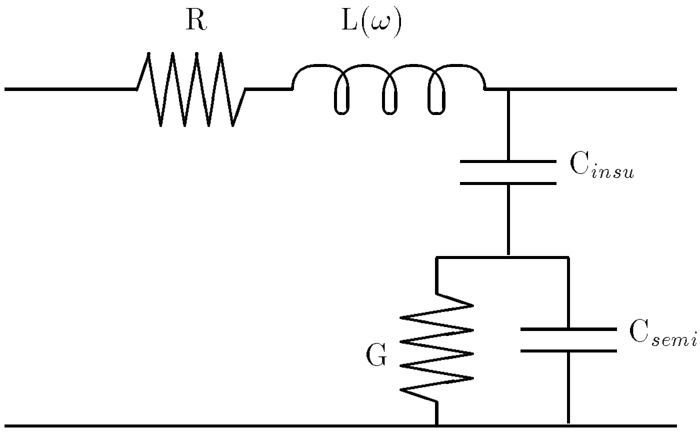

Figure 2.4: Circuit model for a MIS microstrip line, C$_{semi}$ and C$_{insu}$ are the capacitances representing semiconductor and insulator layers respectively.

ratio), for $k = 3h + w/2$ (Fig. 2.3).

The equivalent circuit used to find the admittance of the microstrip is the one proposed in [14] (Fig. 2.4): a capacitor C_{insu} stands for the dielectric layer, in series with a conductance shunted by another capacitance accounting for the semiconducting layer. The circuit model is useful for the calculation of propagation parameters. The same circuit model can be used in common circuit simulators after a proper lumping algorithm [24]. The shunt conductance scales identically with the shunt capacitance in the semiconducting layer. Here, Wheeler's equations are used to find the quasi-static capacitance of the semiconductor substrate, and the value of the conductance obtained using

$$G_{semi} = \frac{\sigma}{\epsilon_{semi}} C_{semi} \tag{2.12}$$

where ϵ_{semi} is the dielectric constant of the semiconductor. The capacitance introduced by the top insulating layer is assumed to be the parallel plate capacitance since the line width is normally much larger than the thickness of the insulating layer, and is given by,

$$C_{insu} = \frac{\epsilon_{insu}}{t} w \tag{2.13}$$

where t is the thickness of the insulating layer.

2.3.2 Comparison with Full-wave Calculations

To evaluate the accuracy of the approach explained above, the complex propagation constants of two microstrip line structures have been obtained using full-wave and transverse resonance technique calculations. The spectral domain approach was used for the full-wave calculations[20]. To compare the results from the full-wave and model, a surface is generated by the model and plotted on the same graph with the full-wave calculations.

As the first example, the case that Hasegawa [14] used originally, and referenced by others later, is chosen. The geometry of the Metal-Insulator-Silicon (MIS) structure is as follows: the width of the microstrip line is 160μm, on a 1μm silicon dioxide layer on a 250μm thick silicon substrate. In the literature, curves of complex propagation constant of the structure with respect to either conductivity or frequency are available. Here, a surface is plotted to examine changes with both variables. Figure 2.5 shows the attenuation constant versus frequency and conductivity. The solid lines are the full-wave calculations. The model predicts the behavior of the attenuation constant quite well. For each operating frequency value, there exist a minimum attenuation corresponding to a conductivity value. For this case the minimum attenuation occurs at 700$(\Omega\text{mm})^{-1}$. As the conductivity increases or decreases, the attenuation increases. Thus, it is difficult to state intuitively what should be the substrate conductivity for minimum attenuation.

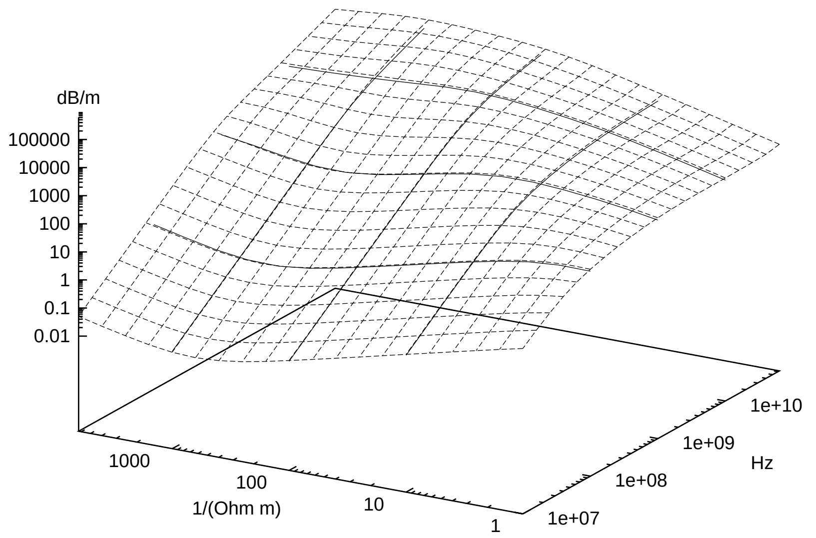

Figure 2.5: Surface of attenuation constant α, versus conductivity and frequency for microstrip line; dotted lines: quasi-static model discussed in the text; solid lines: full-wave results.

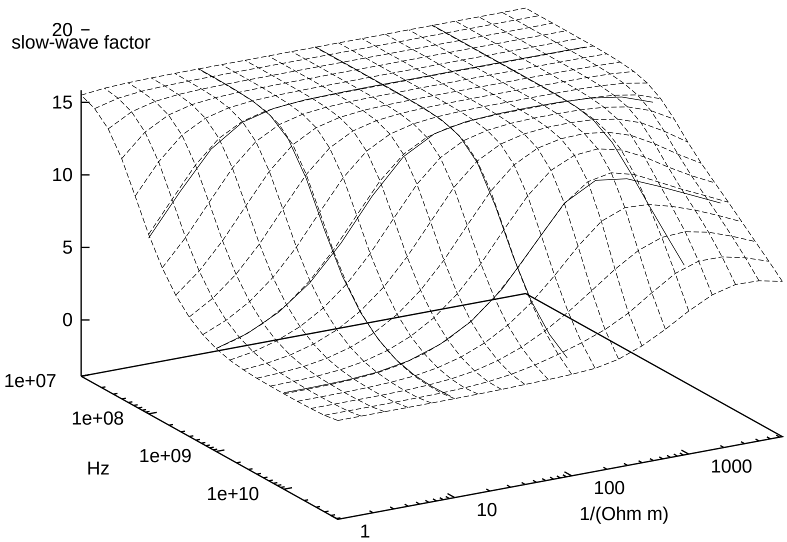

Figure 2.6: Surface of slow-wave factor β/β_0 versus conductivity and frequency for microstrip line; dotted lines: quasi-static model discussed in the text; solid lines: full-wave results.

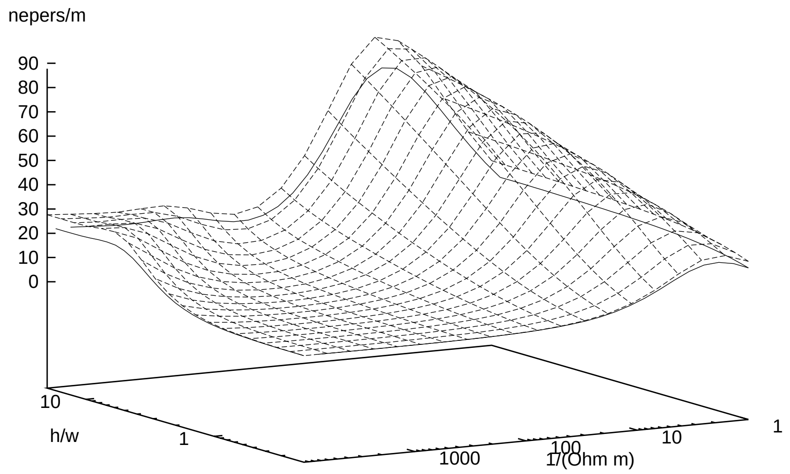

Figure 2.7: Attenuation constant (in nepers per meter) versus height-width ratio of microstrip and conductivity of substrate; dashed net is the quasi-static model and solid lines at borders are full-wave calculations.

As for the phase constant curve, the model agrees with the full-wave calculations to a great accuracy. In the intermediate region where the mode is none of the three modes described in [14] exactly, the model has a sharper transition, and then it stays lower than the full-wave calculation (around a conductivity of 10 $(\Omega\mathrm{cm})^{-1}$ and frequency of 10GHz). A possible cause for this discrepancy may be the field distribution in the highly conductive substrate can not be approximated by quasi-TEM analysis. Although not highly accurate over this small region, the model has the same trend over the whole range of frequency and conductivity with full-wave calculation.

As the second example, the h/w dependence of the model has been checked for a range of conductivity values, keeping the frequency constant at

1GHz. For this example the line width is 50μm. The thickness of the silicon dioxide layer is 1μm, and the thickness of the silicon layer is changing from 10μm to 1000μm. In Fig. 2.7 attenuation constant is plotted. The solid lines are the full-wave curves. The high value of the attenuation constant corresponds to the region where the mode is switching from dielectric quasi-TEM to slow wave mode. Outside of that region the attenuation constant stays small, since the lossless ground plane is shorting out the silicon layer. The mode is slow except at very high conductivity and height/width ratio where it is skin-effect mode. In the skin-effect mode region, at a fixed frequency the surface impedance of a conductor decreases with the square root of increasing conductivity. That is not the case for microstrip transmission lines, since the fields assume a trapezoidal-like shape in the transverse plane, and the area of this shape is proportional to the square of the thickness. In the skin-effect region the thickness can be approximated by the skin-depth. The resistance of this cross section in the propagation direction then can be calculated as independent of the conductivity.

These results were calculated on an IBM RISC/6000 workstation. It took 0.25s to calculate a single quasi-static curve (not the surface). Using the full-wave approach, it took over 1000s to calculate the same curve. Considering the accuracy obtained and computer time taken to reach that accuracy, this model is very efficient.

In this chapter a quasi-static model for a microstrip on a semiconducting substrate has been verified and proven to be efficient for the calculation of propagation parameters. This approach should be very useful in interactive CAD and development tools. The model can be extended to multi layer substrates with arbitrary parameters, i.e., different permeability, permittivity, and conductivity, using transmission line techniques. Even for the high frequency, high conductivity case, the mode is still quasi-TEM. The transverse dimensions over which the currents are distributed decreases as frequency increases, hence the wavelength is always greater than the transverse dimensions. The frequency at which the quasi-TEM assumptions breaks down is in fact much higher than predicted by looking at the transverse physical dimensions of the structure. The quasi-TEM approximation is valid for lossy substrates for a very wide range of frequency and conductivity.

This approach of calculating loss using transverse resonance technique can be extended to conductor loss calculations for lossy transmission lines in homogeneous and lossless media. Geometrical partitioning of rectangular cylindrical conductors in the transverse plane can lead to an approximation for the impedance seen by the traveling wave in the structure at the surface of the conductors. From this information frequency dependent inductance and resistance can be calculated.