Chapter 3

Conductor Loss Calculation of Coplanar Waveguide

3.1 : Introduction

Characterization of coplanar waveguides (CPW) on a lossless substrate is important in understanding base line CPW behavior, especially the conductor losses in both the low and high frequency regimes. Conductor loss evaluation still remains one of the more difficult problems in transmission line analysis. To calculate the conductor loss in a CPW configuration, a new quasi-static approach is shown in this chapter that uses a conformal mapping technique. This novel technique uses no fitting parameter to match the experimental results: only the dimensions, metal conductivity, and the substrate dielectric constant are required to accurately predict the complex propagation constant. This method takes into consideration both the skin effect and current crowding in the conductors. It is also numerically efficient when compared with some fullwave calculations [Heinrich 1990] to measure the conductor loss. Later in this chapter, this technique is compared with other existing quasi-static approaches to show the clear superiority of this method. One of the advantages of this technique is that the methodology can easily be applied to other transmission line structures, such as co-axial cables, twin-lead, parallel square bars, microstrip lines, or co-planar strip lines to calculate conductor losses [Tuncer et al. 1994]. Traditional approaches such as incremental inductance rules [Ramo et al. 1984] or conventional conformal mapping techniques [Collin 1992] fail to adequately model the transition from low frequency to high frequency behavior, especially for small dimension (micron size) interconnects. When modeling the propagation of broad bandwidth time-domain pulses, such as in digital systems, the dispersion induced by this transition can be very significant. Thus, there is a need for models that accurately predict both the attenuation and phase constants for transmission lines using conductors with finite conductivity. These problems are especially acute in
planar transmission lines, such as coplanar waveguide, because the current crowding at the edges of the conductors can lead to a significant increase in ohmic losses.

In this chapter, a brief discussion of traditional quasi-static approaches for calculation of conductor losses and associated phase constants is followed by the detailed description of the new quasi-static model for analyzing transmission lines. Furthermore, it is shown that this novel technique, which is based on conformal mapping, provides excellent agreement between modeled and measured conductor loss over a wide range of dimensions and metal thicknesses. Preliminary results, predicted from this model, are also given for a high T_c superconductor CPW.

3.2 : Issues concerning conductor loss

There are two dominant effects that determine the frequency-dependent current distribution, and hence the ohmic loss, in conductors. The first effect, known as the skin-effect, arises when skin-depth (frequency dependent) becomes smaller than conductor thickness. Skin depth, which is the penetration depth of electric fields into conductors, has an inverse square root relationship with frequency. In addition, when the internal impedance of an isolated plane conductor with a finite conductivity is calculated [Ramo et al. 1984], the internal impedance per unit length and per unit width (which is defined as the surface impedance) can be expressed as

\[ Z_s = R_s + j \sigma \delta = \frac{1 + j}{\sigma \delta} \sqrt{\frac{\pi f \mu}{\sigma}} (1 + j) \]  

(3.2.1)

where, skin depth, \( \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \)  

(3.2.2)

and \( R_s \) is surface resistance and \( L_s \) is the surface inductance. External inductance, arising from fields outside of the conductors, needs to be added with this inductance to get the the full inductive term. Skin effect is an intrinsic property of a conductor, determined only by frequency and geometry, conductivity, permeability, and permittivity of the isolated conductor.
The second dominant effect, known as the proximity effect, arises from current crowding induced by the interaction between the conductors [Kennelly et al. 1915]. At low frequencies, current will be uniformly distributed along the conductor, while at higher frequencies, because of proximity effect, current concentrates more at the edges, adjacent to other conductors owing to the alternating magnetic flux from one penetrating the other. Therefore, to accurately predict the frequency dependent behavior of a transmission line, the model should account for uniform distribution of current at low frequencies, while at higher frequencies both skin depth and proximity effects will redistribute the current to the surfaces of the conductors.

The proposed model for calculation of conductor loss for a planar transmission line uses an isolated-conductor surface impedance in conjunction with a conformal map for the entire transmission line cross-section to account for both skin-depth and current crowding effects.

3.3 : Use of conformal mapping and scaled surface impedance

3.3.1 : Theory

Quasi-static analysis, in particular the use of conformal mapping, is well established as a useful technique for the calculation of propagation constants of quasi-TEM transmission lines. The application of conformal mapping to the calculation of the complex internal impedance of conductors is less well known. Collin uses a conformal mapping technique to obtain the high frequency current distribution in CPW, but uses a standard perturbation technique to approximate the ohmic loss [Collin 1992].

In this new quasi-static formulation, the conformal mapping technique is used to find not only the normal capacitance per unit length of the transmission line, but also to find the frequency dependence of the series inductance and resistance. In general, a conformal map \( w(z) \) (where \( z = x + jy \) is the domain containing the original transmission line) is considered which produces parallel plates in the \( w(u, v) \) mapped plane, with the plates parallel to \( u \)-axis, extending from 0 to \( u_o \), the top plate
located at \( v_t \), and the bottom plate at \( v_b \). The scale factor \( M \) relating a differential length in the \( z \)-plane to one in the \( w \)-plane is just the magnitude of the total derivative of the mapping function:

\[
M(u, v) = \left| \frac{dw}{dz} \right|_{u,v}.
\] (3.3.1)

If the conductors in the original domain are much thinner than their lateral extent, they can be approximated as thin sheets with surface impedance \( Z_s \) given by [Kennelly et al. 1915]

\[
Z_s(\omega) = \frac{\sqrt{\frac{\omega \mu_o}{2\sigma}}(1 + j)}{\tanh\left\{ \sqrt{\frac{\omega \mu_o \sigma}{2}} \left( \frac{t}{2} \right)(1 + j) \right\}}.
\] (3.3.2)

where \( t \) is the thickness and \( \sigma \) the conductivity of the conductors. Equation (3.3.2) describes the normal skin-depth limited surface impedance at high frequency as well as the appropriate low frequency limit, i.e., as \( \omega \to 0 \), \( Z_s \to 2/\sigma t \) (the factor of two appearing because this resistance forms only one side of the plate). This surface impedance can be transformed into the mapped domain using the scale factor \( M(u, v) \), as illustrated in Fig. 3.1. The series impedance \( \delta Z \), due to a differential width \( du \) of the plates, is the series combination of the mapped surface impedance and the parallel plate inductance:

\[
\delta Z = \left\{ Z_s M(u, v_t) + Z_s M(u, v_b) \right\} + \frac{j \omega \mu_o |v_t - v_b|}{du}.
\] (3.3.3)

For "thin" plates (or any other conductor with rotational symmetry), surface impedance is independent of position on the conductor surface, therefore, eqn. (3.3.3) can be simplified as
\[ \delta Z = \frac{Z_S \{M(u,v_t) + M(u,v_b)\}}{du} + \frac{j \omega \mu_o |v_t - v_b|}{du}. \] (3.3.4)

Fig. 3.1: Diagram illustrating use of a conformal map to find the series impedance of a transmission line including the effect of finite resistance.

The total series impedance per unit length, including the impact of finite resistance, is then found from the parallel combination of the impedances of each differential width of the plates:

\[ Z(\omega) = \left[ \int_0^{u_o} \frac{du}{j \omega \mu_o |v_t - v_b| + Z_S \{M(u,v_t) + M(u,v_b)\}} \right]^{-1}. \] (3.3.4)

At low frequencies and when \( Z_s \) is real, the impedance, \( Z(\omega) \) reduces to

\[ Z = R_{dc} + j \omega \mu_o |v_t - v_b| \left( \frac{R_{dc}}{Z_S} \right)^2 \int_0^{u_o} \frac{du}{\left[ M(u,v_t) + M(u,v_b) \right]^2}. \] (3.3.5)
where $R_{dc}$ is the dc resistance of the original conductors. At high frequencies, Eq. (3.3.3) becomes

$$Z = \frac{Z_S}{u_o^2} \left\{ \int_{u_o}^{u_o} du \left[ M(u, v_t) + M(u, v_b) \right] \right\} + j\omega L_{ext} \quad (3.3.6)$$

where $L_{ext}$ is the normal external inductance of the line (given by $L_{ext} = \frac{\mu_o}{u_o} |v_t - v_b|$ in the mapped domain).

### 3.3.2 : Scaled conductivity and a simple example

As a simple example of using the conformal mapping technique to calculate the series impedance through scaled conductivity in the mapped domain, a cylindrical wire is considered. In this case, conductivity is scaled in the mapped domain to calculate the surface impedance instead of using scaled surface impedance. A differential length in real space (i.e., in x-y plane) is scaled by an amount $M(u, v)$ in the mapped domain (i.e., in u-v plane), where $M(u, v)$ is defined as in eqn. 3.3.1. Therefore, $dx$ and $dy$ in real space scale into $Mdx (= du)$ and $Mdy (= dv)$ respectively in the mapped domain. For the cylindrical wire case, simple mapping function turns the wire into a conducting rectangular slab with non-uniform conductivity, as shown in Fig. 3.2. The differential lengths in real space ($dr$ and $rd\theta$) become $du$ and $dv$ respectively in the w-plane, where $r\partial\theta = \partial v/M$ and $r\partial r = \partial u/M$. Therefore to calculate the shunt capacitance, which depends on the ratio of side lengths, the scale factor cancels as

$$\partial C \propto r \frac{\partial \theta}{\partial r} = \frac{(\partial v/M)}{(\partial u/M)} = \frac{\partial v}{\partial u}. \quad (3.3.7)$$

But to calculate the series resistance in the mapped domain, which depends on the area, the scale factor enters as $M^2$ as

$$\partial R = \frac{1}{\sigma} \frac{1}{r \partial r \partial \theta} = \frac{1}{\sigma} \frac{1}{(\partial u/M)(\partial v/M)} = \left( \frac{M^2}{\sigma} \right) \frac{1}{\partial u \partial v}. \quad (3.3.8)$$
Therefore, effective conductivity (scaled) in the mapped domain depends on the scale factor as

\[ \sigma_{M}' = \sigma_{M} \cdot r \]

\[ \sigma_{M}(\text{scaled}) = \frac{\sigma}{M^2} \quad \text{(3.3.9)} \]

With this scaled conductivity in the mapped domain, the mapped surface impedance can be found using non-uniform transmission line equations [Tuncer 1993], and the result is

\[ Z_s = -\frac{T J_0(T_{r0})}{2\pi r_{r0} \sigma J_0'(T_{r0})} \quad \text{(3.3.10)} \]

where \( T^2 = -j\omega\mu\sigma \), \( J_0 \) is the Bessel function of zeroth order, and \( ' \) (prime) stands for first differentiation of the function. In the real domain for the circular cylindrical wire, the Helmholtz equation can be solved directly too [Ramo et al. 1984], where exactly the same analytic expression is found for surface impedance. For the above example, calculating the surface impedance from the mapped plane is harder than it is in the real plane. Therefore, one does not gain by conformally mapping the structure to analyze it. This example verifies that this technique is good enough, or in certain cases, exact enough, to calculate the series impedance. For structures like co-planar strip line or co-planar waveguide, this methodology is much better than some...
existing techniques to calculate the series impedance, and hence the conductor loss, both from accuracy and numerical efficiency viewpoints.

3.2.3 : Formulations of code

To formulate the co-planar waveguide conductor loss calculation based on conformal mapping, three main steps are involved. In the first step conductors are conformally mapped into a parallel plate configuration. In the second step, very small patches of conductors in the real plane are considered and the surface impedances of corresponding small patches in the mapped plane (scaled surface impedances) are calculated where both skin effect and current crowding effect are taken into account. In the third and final step, the contribution from each infinitesimal patch is added in parallel to calculate the total impedance, from which the propagation constant of the transmission line is calculated.

Conformal mapping is performed in three steps as explained in [Wentworth et al. 1989]. The final step is to evaluate an elliptic integral with two singular points, one at the center conductor edge and the other at the ground plane edge. Integration is performed along the surface of the substrate material (which has two singular points) and along the metal surface of the conductors, which has no singular points because of finite thickness from the axes chosen. The x-axis is chosen along the top surface of the substrate and the y-axis along the vertical line passing through the mid-point of the center conductor (because of symmetry, only one half of the CPW is considered for mapping and subsequent calculations). For simplicity in calculation and for avoiding two singular points, integration along the surface of the substrate is performed in three steps; first, from the center of the axes (mid-point of the center conductor) to a point which is very close of the edge of center conductor; second, an integration is done for the gap region; third, an integration is performed along the ground plane edge starting immediately after the start of ground plane to the other edge of the conductor. These integration steps can be shown symbolically as

\[
\int_0^{a+s+w} \int_0^{a-s+\varepsilon} \int_{a+\varepsilon}^{a+s+\varepsilon} = \int_0^{a-s+\varepsilon} \int_{a+\varepsilon}^{a+s+\varepsilon} + \int_{a+\varepsilon}^{a+s+\varepsilon} \int_{a+\varepsilon}^{a+s+\varepsilon} + \int_{a+\varepsilon}^{a+s+\varepsilon} \int_{a+\varepsilon}^{a+s+\varepsilon} (3.2.11)
\]
where \( a, s, w \) are half center conductor width, gap between the center and ground plane, and ground plane width, respectively, and \( \varepsilon \) stands for a very small number in the range of 0.01\% to 0.03\% of center conductor width. Integration is done with the 24-point gaussian quadrature rule, where even with the moderate number of points considered, both accuracy and numerical efficiency of the integration is reasonable. Mapped domain CPW conductors with a finite ground plane width and with a finite thickness of the substrate preserve the parallel plate configuration with non-rectangular shaped conductors, as shown conceptually in Fig. 3.3.

Fig. 3.3 : Conformal mapping of a rectangular-shaped coplanar line based on Schwarz-Christoffel transformation (drawn not to scale).
After the mapping is performed, the conductors (both center and ground) are subdivided into equal numbers of small patches in the real plane and the corresponding patches are identified in the mapped domain. The surface impedance is then calculated for each small "parallel" subsection of the center conductor and ground plane. Surface impedances of each individual small patch for both center conductor and ground planes are then added in series with the external inductances to get a full impedance. The total impedance of the structure can be found by adding the impedances of the patches in parallel. This total impedance can be used to calculate the propagation constant of the line. At high frequencies in the mapped domain, the current distribution is uniform along the conductors, but the shape is deformed to accommodate the unequal current distribution in real space. In addition, skin effect is considered in calculating impedances of each small patch in the mapped domain, and therefore, all the effects involving the conductor loss are considered in this method. In contrast, at low frequencies the current distribution is uniform in real space, but not in the mapped domain. To verify the methodology and the accuracy of the integration, the total resistance of the transmission line at dc (zero frequency) and the inductance at high frequencies were found to be in good agreement with expected results. Furthermore, as shown in Section 3.5, the frequency dependence of conductor loss is in good agreement with our experimental results, whereas some other existing quasi-static calculations fail to match. This methodology is also numerically efficient. The formulated code used to calculate conductor losses was run on a Sun workstation and took only 7.5 seconds to complete the mapping and 6.5 seconds to perform the subsequent calculations, using non-optimized Fortran code.

3.4: Other conductor loss calculations

The conductor loss of a planar structure like a co-planar waveguide can be calculated by quasi-static or fullwave approaches. Previous fullwave studies [Heinrich 1990] show that a co-planar waveguide has low dispersive propagation
characteristics, suggesting that it can be modeled accurately even from a quasi-static perspective, provided a proper model is chosen.

The incremental inductance rule is applicable only when the conductor thickness is several times the field penetration depth. On the other hand, a complete dc calculation considers uniform current distribution along the conductors with full field penetration, and therefore, ignores both current crowding and skin effects at high frequencies. Furthermore, the standard incremental rule, both analytical [Gupta et al. 1979] and numerical [Wheeler 1977], is not sufficient to predict the results involving the frequency dependence of both current crowding effect at the edges and skin effect along the surfaces of conductors. Therefore, a composite model is essential in predicting both the low and high frequency behavior of a planar transmission line. A number of loss analyses have been performed for microstrip, strip line and co-planar waveguide (CPW) structures. Some early numerical calculations [Davies et al. 1977; Mirshekar-Syahkal 1979] were performed for coplanar waveguide conductor loss, but with a limited range of parameters. Gopinath used a quasi static method [Gopinath 1982] to calculate the conductor loss by using the longitudinal current distribution pattern without the dielectric substrate to get the charge distribution in the CPW conductors. To calculate the attenuation constant, it is usually assumed that the fields in the no-substrate case (lossless) are the same as the fields with a dielectric substrate. This may lead to erroneous results for planar structures with sharp edges [Pregla 1980] (as with coplanar waveguide). Gupta's [Gupta et al. 1979] formula for coplanar waveguide conductor loss is only an analytical form of the incremental inductance rule. Lee et al. [Lee et al. 1989] proposed a phenomenological approach for calculating conductor loss of a planar quasi-TEM transmission line. This phenomenological loss equivalence method (PEM) is based on the change in current distribution as the field penetrates into the transmission lines. In this approach, a single strip is approximated to replace the transmission line, where both have the same conductor loss. The internal impedance of the single strip is then calculated and put into the transmission line model to get all the propagation characteristics. The authors also extended this method to calculate the conductor loss of a high-T \textsubscript{C} superconductor. Heitkamper et al. [Heitkamper et al. 1991] calculated the conductor loss by means of the standard perturbation approach.
They started from a lossless waveguide analysis and calculated the losses from the corresponding surface currents on the conductors, assuming the tangential magnetic field remained approximately the same for lossless and lossy cases. Heinrich has performed a number of conductor loss calculations involving both quasi-static and fullwave analyses. He has done a computationally involved mode-matching technique [Heinrich 1990] to calculate the conductor loss of a planar structure. He observed significant deviations for CPW loss when compared to the conventional assumption of lossless zero-thickness strips. Heinrich also used a quasi-TEM analysis [Heinrich 1993] of a coplanar transmission line to calculate the conductor loss over a wide range of frequency of operation, but performed piecewise analysis to match experimental conductor loss calculations with his modeled results. Alessandri et al. [Alessandri et al. 1992] has performed conductor loss analyses for thin MMIC coplanar lines with modest computer effort where he used modified perturbational methods in conjunction with a transverse resonance technique. Collin [Collin 1992] used conformal mapping and standard perturbational method to calculate the ohmic losses of coplanar transmission lines. He used the conventional approach based on an integral over the high frequency current density in the real space domain which he assessed through conformal mapping, and then evaluated the integral along the surfaces of the conductors in the real plane. However, this is valid only in the high frequency regime and therefore, his conductor loss calculations are in good agreement with experimental results only at very high frequencies and do not apply for $R_{dc} > \omega L_{ext}$.

3.5 : Model calculation and comparison

To verify the accuracy of our new technique, measurements have been made on coplanar waveguide fabricated on low loss dielectric substrates. Two different dielectric substrates, semi-insulating GaAs and glass (Pyrex), were used for rf measurements from 45 MHz to 40 GHz. Two different metallization materials were also used, 0.8 µm of evaporated silver on top of the GaAs substrate, and 0.8 µm of electro-plated gold on the Pyrex sample. The CPW dimensions are shown in Fig. 3.4. The conformal mapping functions discussed in [Wen 1969; Wentworth et al. 1989] were used in Eq. 3.2.3.
Both the experimental and the simulated results from this model are shown in Figs. 3.5 and 3.6. For the experimental results, attenuation and effective refractive index \( n_{\text{eff}} = \beta / \beta_0 \) were extracted from the S-parameters measured by an HP 8510B Network Analyzer. Excellent agreement between the experimental results and the calculations for both samples over the full frequency range (45 MHz - 40 GHz) was obtained. The effective resistance (the real part of Eq. 3.3.6) and inductance (from the imaginary part of Eq. 3.3.6) calculated from our model for these two structures are shown in Figs. 3.7 and 3.8. Also, the calculated attenuation constants from existing quasi-static conductor loss analyses [Wheeler 1977; Gupta et al. 1979; Collin 1992] with the experimental results are shown in Fig. 3.9.

Fig. 3.4: Cross-sectional drawing of Coplanar waveguide; Half of center conductor width, \( a = 5 \mu m \), gap between center conductor and ground plane, \( (b-a) = 7 \mu m \), ground plane width, \( w = 500 \mu m \), and conductor thickness, \( t = 0.8 \mu m \).

To further verify the applicability of this technique, we have also compared the extensive experimental data of Haydl et al. [Haydl 1992; Haydl et al. 1992] to our model; in all cases excellent agreement has been obtained. Figures 3.10 and 3.11
show typical comparisons between their results and our calculations over a wide range of CPW.

Fig. 3.5(a): Comparison between experimental and predicted results for attenuation constant with semi-insulating (SI) GaAs substrate.
Fig. 3.5(b) : Comparison between experimental and predicted results for attenuation constant with pyrex substrate.

Fig. 3.6(a) : Comparison between experimental and predicted results for effective index of refraction ($n_{eff}$) with semi-insulating (SI) GaAs substrate.
Fig. 3.6(b) : Comparison between experimental and predicted results for effective index of refraction ($n_{\text{eff}}$) with pyrex substrate.
Fig. 3.7: Variation of series resistance and series inductance with frequency for pyrex substrate.
Fig. 3.8: Variation of series resistance and series inductance with frequency for semi-insulating GaAs substrate.
dimensions. Again, no fitting factors are used; only the dimensions of the CPW and conductivity of the metal are required for the calculation.

**3.6 : Conductor loss calculations for high T<sub>C</sub> superconductors**

Conductor loss calculations based on conformal mapping techniques can also be extended to high T<sub>C</sub> superconductor planar structures, like coplanar waveguide (CPW). Conformal mapping based conductor loss calculation uses the conductivity and hence the surface impedance in the real plane, which can be evaluated for a superconductor using the two-fluid method [Duzer et al. 1981]. The surface impedance calculated in this method is then utilized in the mapped domain of our conductor loss model to calculate total series impedance and hence the conductor loss of a superconductor in the microwave and millimeter-wave regime.

Fig. 3.9 : Comparison of conductor losses between experimental results and those calculated from other existing quasi-static techniques.
In the two-fluid method, electrons are considered to be divided into two types, e.g., normal electrons and superconducting Cooper pairs. These two sets of electrons produce the overall complex conductivity of the superconductor, as shown below [Kong 1991]:
Fig. 3.11: Comparison of predicted conductor losses with the extensive experimental work done by Haydl [Haydl et al. 1992]. The plot shows attenuation constant versus frequency for different CPW geometries and conductor thicknesses on InP substrate; $a$ stands for half of center conductor width, $(b-a)$ for gap width and $t$ for conductor thickness. All dimensions given in the plot are in microns.

$$\sigma_{sc} = \sigma_1 - j\sigma_2 = \sigma_n \left( \frac{T}{T_c} \right) - j \left( \frac{1 - \left( \frac{T}{T_c} \right)^4}{\lambda_o^2 \omega \mu_o} \right)$$

(3.6.1)

where $T$, $T_c$, $\sigma_n$, and $\lambda_o$ are the operating temperature, critical temperature, conductivity at $T_c$, and penetration depth at DC and at 0 K, respectively. It is to be noted that the imaginary part of $\sigma_{sc}$ has an inverse relationship with the frequency. From this complex conductivity, the surface impedance can be calculated as
\[ Z_s = R_s + jX_s = \sqrt{\frac{j\omega \mu_o}{\sigma_1 - j\sigma_2}}. \] (3.6.2)

From the above equation, \( R_s \) and \( X_s \) can be calculated as [Kong 1991]

\[ R_s^2 = \frac{\omega \mu_o \sigma_2}{2(\sigma_1^2 + \sigma_2^2)} \left( \sqrt{\left( \frac{\sigma_1}{\sigma_2} \right)^2 + 1} - 1 \right) \] (3.6.3)

\[ X_s^2 = \frac{\omega \mu_o \sigma_2}{2(\sigma_1^2 + \sigma_2^2)} \left( \sqrt{\left( \frac{\sigma_1}{\sigma_2} \right)^2 + 1} + 1 \right). \] (3.6.4)

Here, if \( \sigma_2 \gg \sigma_1 \), then \( R_s \) becomes zero and also at lower temperature \( R_s \) decreases, in both cases causing stronger superconductivity.

Fig. 3.12 : Calculated attenuation constant and effective series resistance (real part of series impedance) of a high Tc superconductor CPW. CPW dimensions are center conductor width \( 2a = 10 \mu m \), gap \( b-a = 7 \mu m \) and ground plane width \( w = 500 \mu m \).
The two-fluid method is used in this calculation only because of its simplicity and reasonable agreement with experimental data, although it requires some additional terms (such as residual resistance [Duzer et al. 1981]), to fit the model data with experimental results.

In calculating conductor loss for high $T_c$ material, based on conformal mapping, YBa$_2$Cu$_3$O$_{7-x}$ is considered, whose $T_c$ is taken as 92.5 K. Normal conductivity at $T_c$ is taken as 1.0 S/µm and penetration depth (at 0 K and at DC), $\lambda_0$, is assumed to be in the range of 0.15 µm - 0.45 µm [Kong 1991]. The operating temperature is considered as 77 K, which is the boiling temperature of liquid nitrogen. The surface resistance is first evaluated and using this surface resistance, conductor loss is calculated, which is shown in Fig.3.12. In the same figure, the real part of series impedance ($R_{\text{eff}}$) is also shown. This calculation demonstrates that the conformal mapping methodology of calculating conductor loss can be applied to high $T_c$ materials as well as with normal conductors.

3.7 : Summary

In this chapter, modeling of co-planar waveguide conductors is presented. The model is based on quasi-static conformal mapping of the CPW structure. The non-uniform current distribution is transformed into a uniform shape in the mapped domain, where conductivity becomes irregular. The mapped conductor surfaces are then analyzed and a scaled surface impedance is calculated, from which the total impedance is evaluated. The model results show an accurate match with experimental results.