2. Transmission Lines

Introduction

Transmission lines are electrical structures that are significantly larger in one dimension than the others (length). Unlike lumped elements, the voltage, current, and phase varies along the length. The voltage and current on the transmission line also depend on the terminating impedance at the far end and the output impendence of the circuit feeding the transmission line.

Microstrip transmission lines will be the primary focus of this dissertation. The method below could be extended to include other examples of transmission lines such as coplanar waveguides, coplanar strips, rectangular waveguide, coaxial cable, and twin lead.

Propagation Constant and Characteristic Impedance

Transmission lines are usually characterized by two complex numbers: propagation constant and characteristic impedance. All four quantities (i.e., the real and imaginary parts of S_{11} and S_{21}) are needed to fully characterize a line, and the quantities can be and often are functions of frequency.

The propagation constant, γ, describes the behavior of a signal as it propagates down a transmission line. The real part of γ is usually called the attenuation constant, α. The attenuation constant describes how much the magnitude of a signal is reduced as it propagates and is defined as a positive quantity for any passive line because negative values indicate gain. The imaginary part of the propagation constant is β, the phase constant. As the name implies, it describes how the phase changes as a signal propagates. It is also defined as a positive number because negative values would indicate propagation in the opposite direction.

The characteristic impedance, Z_0, relates the voltage to the current on the line as a signal propagates. The real part, R_0, is assumed to be positive, but the imaginary part, X_0, can be of either sign. The characteristic impedance in the high frequency limit is often described as a single frequency independent real number because at high enough frequency, the imaginary part of the characteristic impedance is zero, and the real part is a frequency independent, non-zero number.

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A transmission line can be divided into infinitesimal sections (which are small compared to a wavelength), each with a lumped circuit interpretation, as shown in Figure 2.1. The per unit length parameters are the series resistance R, the series inductance L, the shunt capacitance C, and the shunt conductance G. The advantage of the circuit interpretation is the frequency independence of the RLCG parameters under certain frequency, geometrical, and material constraints. For a microstrip test structure with conductor thickness less than a skin depth over the frequency range of interest, R, L and C will be frequency independent. The RLCG parameters can be related to the transmission line parameters as shown in Equations (2.1) and (2.2), where ω is the frequency in radians. Equations (2.1) and (2.2) still hold even if the RLCG parameters are frequency dependent.

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{2.1}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{2.2}$$

Some interesting observations can be made from Equations (2.1) and (2.2). At large enough ω, ωL will dominate R and ωC will dominate G. In this limit, Z_0 will be purely real, and γ will be purely imaginary. This is referred to as the high frequency limit, or the lossless limit, and is a common limit used in microwave design and measurement. Several other limits are possible, but the most common is the RC limit which occurs when R is much greater than ωL and ωC is much greater than G. In this limit, the real and imaginary parts of Z_0 are of equal size, but opposite signs and α and β are equal.

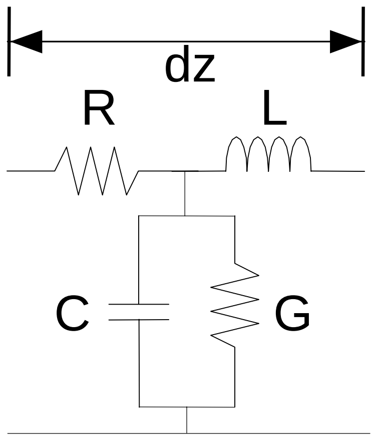

Figure 2.1: Circuit diagram of an infinitesimal length of transmission line.

Loss Tangent

When a microstrip transmission line is embedded in a uniform lossy dielectric, the current flows along the field lines in the dielectric, so the relation between G and C is simply

$$G = \omega C \cdot \mathrm{Tan}\delta . \qquad (2.3)$$

Hence, G will linearly increase in frequency for a frequency independent loss tangent and capacitance. If a measurement can determine G and C with sufficient precision, the loss tangent can be extracted. The assumption of a frequency independent loss tangent sets the ratio of G to ωC and, therefore, sets the dominant term. By setting ωC as the dominant term, Equations (2.1) and (2.2) will never reach certain limits (an RG transmission line for example).

Measuring Transmission Lines

Transmission lines can be measured in many different ways. Two common methods are impedance spectroscopy (measurement of input impedance) and network analysis (measurement of scattering parameters). Both are steady-state, swept frequency measurements that can determine some or all of the RLCG parameters.

Impedance Spectroscopy

Impedance spectroscopy measures the input impedance of a transmission line as a function of frequency. Impedance analyzers can measure over frequencies ranging for 100 Hz to 1.8 GHz, though a given instrument will likely not cover the entire frequency range.

The measurement of input impedance is a 1-port measurement. This means that only one complex number (Z_{in}) is measured at each frequency. For a transmission line, there are four unknowns (R, L, C, and G), so the system is underdetermined. If the transmission line is in a two variable limit (such as the RC limit), there are 2 unknowns, and the system is sufficiently determined.

The input impedance of a transmission line is

$$Z_{in} = Z_0 \frac{Z_{load} + Z_0 \tanh \gamma\ell}{Z_{load} \tanh \gamma\ell + Z_0} \tag{2.4}$$

where Z_0 and $\gamma\ell$ are defined by Equations (2.1) and (2.2) and Z_{load} is the load at the far (non-measurement) end of the transmission line. The far end is often terminated with an open circuit that, ideally, has an infinite impedance, and the input impedance reduces to

$$Z_{in} = \frac{Z_0}{\tanh \gamma\ell} \ . \tag{2.5}$$

The load impedance is never a true open because there are end effects at the end of the transmission line. For transmission lines manufactured on silicon wafers, microwave probes are needed to measure the lines, and probe pads are needed for the probes. If the transmission line is designed for 2-port measurements, there will be probe pads at both ends. Hence, the termination for a 1-port measurement will be approximately the probe pad capacitance.

Another method of interpreting input impedance data is to assume that the transmission line is in the RC limit and is short enough ($|\gamma\ell| << 1$) to justify using a lumped circuit model. In this case, the transmission line consists of only a lumped resistor and a lumped capacitor, as shown in Figure 2.2. The real part of the input impedance is attributed to the resistance, and the imaginary part of the input impedance is attributed to the capacitance, allowing R_{total} and C_{total} to be determined using

$$R_{total} = \mathrm{real}(Z_{in}) = R \cdot \ell \text{ and} \tag{2.6}$$

$$C_{total} = \frac{-1}{\omega \cdot \mathrm{imag}(Z_{in})} = C \cdot \ell \text{ .} \tag{2.7}$$

The measured quantities are total quantities, not per length quantities, because the assumption has been made that the line is short enough to be treated like the circuit in Figure 2.2. The per unit length quantities can be determined by dividing by length. C_{total} includes any pad capacitance that is present, but the pad capacitance is only important if it is of significant size compared to the total capacitance of the line.

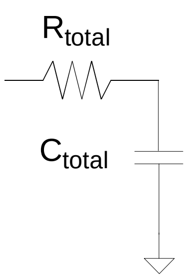

Figure 2.2: Circuit diagram of a lumped circuit equivalent for an electrically short, RC transmission line.

Impedance spectroscopy can be a useful method for characterizing transmission lines, particularly at frequencies less than 100 MHz. The major drawback of impedance spectroscopy is the 1-port nature of the measurement because certain assumptions (lumped RC model) must be true. Fortunately, the frequencies at which the impedance analyzer is most useful are also the frequencies at which the required assumptions are most likely to be true.

Network Analysis

Network analysis involves measuring the scattering parameters (S-parameters) of a network using a network analyzer. S-parameters can completely describe a linear n-port network at its terminals without knowledge of the actual network, so the network can be treated as a black box described only by its S-parameters. The ability to treat the network as a black box is beneficial since all networks, even ones that are nominally well known, contain parasitics that are unknown.

Most commercial network analyzers are able to measure networks with two ports. Networks that have more than two ports can only have two ports measured at a time. The other ports are considered terminated with some impedance. This impedance can be intentional, like an attached 50 Ω load, or unintentional, like an open. The results for the two measured ports depend on the terminations of the unused ports. The results are referenced to the impedance of the measuring instrument.

A block diagram of a 2-port network is shown in Figure 2.3. As the figure shows, a port consists of both a signal and a ground. Four complex S-parameters are necessary to fully describe a 2-port network. Scattering parameters are notated S_{xy}, where x is the port where the measurement is made, and y is the port that is excited. When x and y are the same port (S_{11} and S_{22}), the scattering parameter is a measure of the reflection at the port with the other port terminated in the measurement port impedance. When x and y are different (S_{12} and S_{21}), the scattering parameters are a measure of transmission.

Figure 2.3: Block representation of a 2-port network.

A 2x2 matrix is required to fully describe a 2-port network as shown Equation (2.8).

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{2.8}$$

For a symmetrical and reciprocal network, S_{11} equals S_{22} and S_{21} equals S_{12}. This leaves four quantities (the real and imaginary parts of S_{11} and S_{21}) that can be used to interpret the contents of the network.

The S-parameters for a lossy transmission line are given by Equation (2.9) [27], where Z_{dut} is the characteristic impedance of the transmission line, γ is the propagation constant of the line, ℓ is the length of the line, and Z_0 is the port impedance of the network analyzer.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} =$$

$$\begin{bmatrix} \dfrac{\left(Z_{DUT}^2 - Z_0{}^2\right)\sinh\gamma\ell}{\left(Z_{DUT}^2 + Z_0{}^2\right)\sinh\gamma\ell + 2Z_0 Z_{DUT}\cosh\gamma\ell} & \dfrac{2Z_0 Z_{DUT}}{\left(Z_{DUT}^2 + Z_0{}^2\right)\sinh\gamma\ell + 2Z_0 Z_{DUT}\cosh\gamma\ell} \\ \dfrac{2Z_0 Z_{DUT}}{\left(Z_{DUT}^2 + Z_0{}^2\right)\sinh\gamma\ell + 2Z_0 Z_{DUT}\cosh\gamma\ell} & \dfrac{\left(Z_{DUT}^2 - Z_0{}^2\right)\sinh\gamma\ell}{\left(Z_{DUT}^2 + Z_0{}^2\right)\sinh\gamma\ell + 2Z_0 Z_{DUT}\cosh\gamma\ell} \end{bmatrix}$$

(2.9)

Since transmission lines are symmetrical and reciprocal networks, S_{11} equals S_{22}, and S_{12} equals S_{21}. The S-parameters are implicitly functions of frequency because $\gamma\ell$ and Z_0 may be functions of frequency.

Real networks containing transmission lines have parasitics that are included in the network analyzer measurement, but not in the network model given by Equation (2.9). The S-parameter representation of an assumed parasitic can often be derived, but combinations of networks described by S-parameter matrices cannot be simply combined by multiplying or adding the matrices together. Assuming that the parasitic is in cascade with the transmission line, a different set of parameters, the ABCD parameters, allows the cascading of networks by the multiplication of the

ABCD matrices. Scattering parameters can be converted to ABCD parameters using Equations (2.10)-(2.13) or vice versa using equations (2.14)-(2.17) [28]. The scattering parameters are all referenced to the port impedance, Z_0, of the system to which the network is connected.

$$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}} \tag{2.10}$$

$$B = Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}} \tag{2.11}$$

$$C = \frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}} \tag{2.12}$$

$$D = \frac{(1-S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}} \tag{2.13}$$

$$S_{11} = \frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D} \tag{2.14}$$

$$S_{12} = \frac{2(AD-BC)}{A+B/Z_0+CZ_0+D} \tag{2.15}$$

$$S_{21} = \frac{2}{A+B/Z_0+CZ_0+D} \tag{2.16}$$

$$S_{22} = \frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D} \tag{2.17}$$

Network analysis is useful at frequencies between 40 MHz and 110 GHz, though a given instrument may not cover the entire range. Scattering parameters are useful because they can fully describe a 2- port transmission line without making the assumptions necessary for impedance spectroscopy measurements. Even with the information from 2-ports, the presence of parasitics can increase the number of unknowns beyond the number of knowns.

Summary

Transmission lines can be described by a propagation constant and a characteristic impedance or by a distributed circuit model. In both cases, there are four unknowns (assuming length is known); therefore, a measurement needs to provide 4 knowns. If any of the unknowns can be ignored due the frequency range used for the measurement, fewer knowns need to be provided by the measurement.

Impedance spectroscopy is useful at low frequencies but can only provide two measured quantities. Fortunately, the low frequencies often allow certain parameters for the transmission line to be ignored. For instance, the measurement of input impedance can allow the characterization of RC transmission lines.

Network analysis is useful at high frequencies and allows the measurement of four knowns. The transmission line can be in any limit or in a regime where are all parameters are important. The introduction of parasitics complicates the extraction of the transmission line parameters from the measured S-parameters, but the network model can be expanded to accommodate more unknowns. The expansion of the network model does not increase the number of measured quantities, but other strategies can be used to overcome this underdetermination.