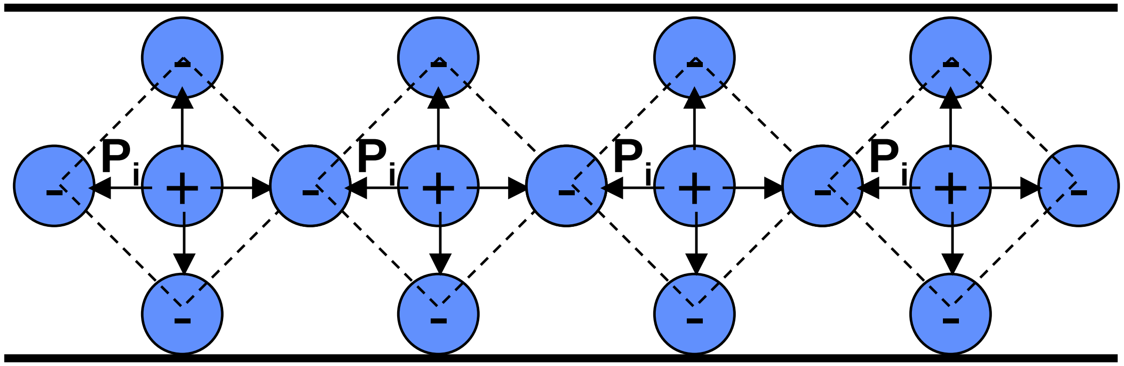
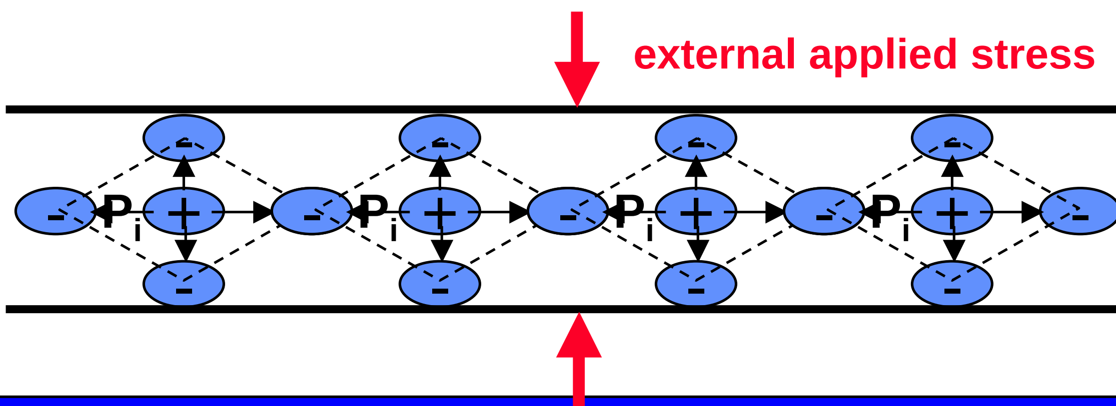
Piezoelectric effect

- depends on polarization, P, in a material
  - symmetry of crystal is critical
  - for crystal with center of symmetry, application of external stress does NOT produce a net polarization



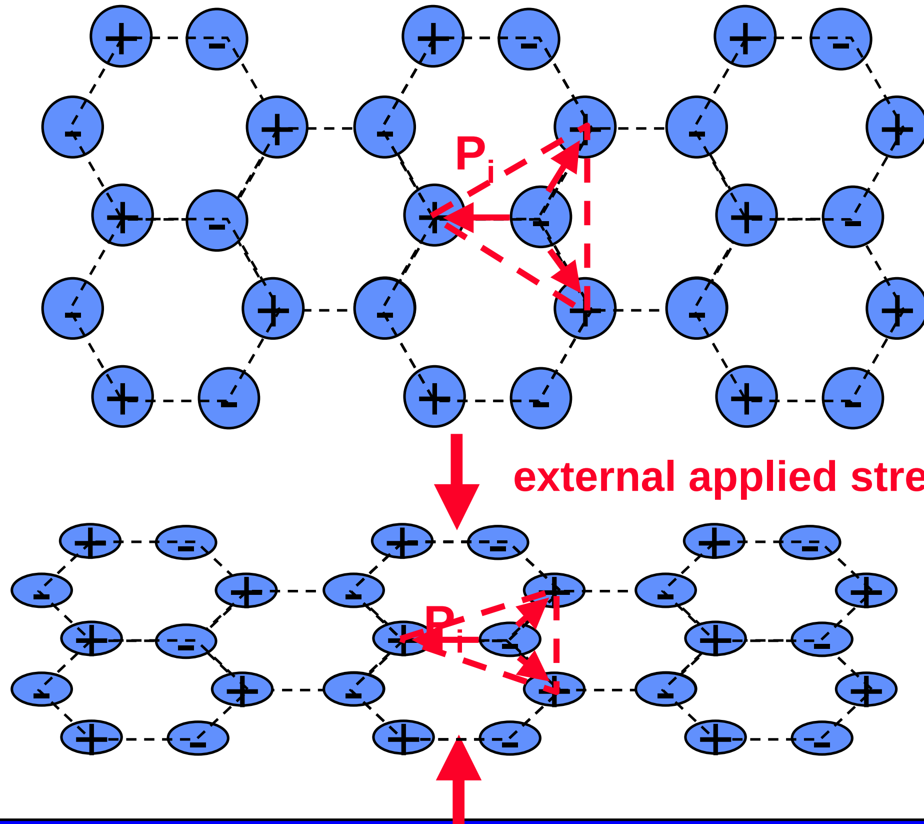
$$\sum \vec{P}_i = 0$$



$$\sum \vec{P}_i = 0$$

Piezoelectric effect

- depends on polarization, P, in a material
  - symmetry of crystal is critical
  - for crystal without simple center of symmetry application of external stress DOES produce a net polarization



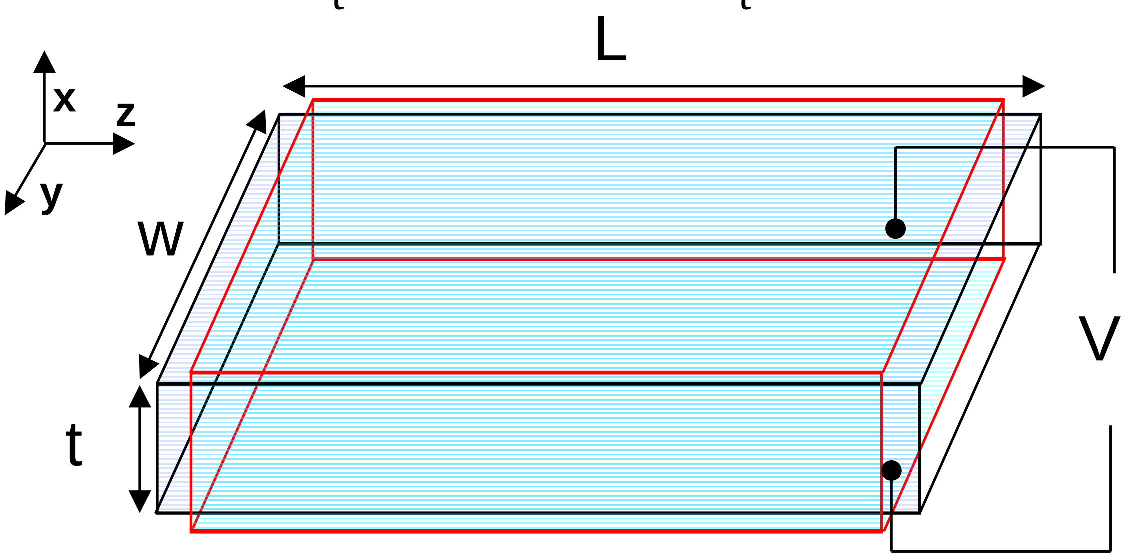
$$\sum \vec{P}_i = 0$$

$$\sum \vec{P}_i \neq 0$$

Piezoelectric effect

- **reciprocity between applied voltage resulting in displacement and applied force/displacement producing voltage**
  - **constants relating voltage to displacement have units of charge/force**

$$\delta L = d_{xz} \cdot V \cdot \frac{L}{t} \qquad \delta W = d_{xz} \cdot V \cdot \frac{W}{t} \qquad \delta t = d_{xx} \cdot V$$



for force applied along any axis, voltage across thickness is

$$V = d_{xz} \cdot \frac{F_z}{\varepsilon \cdot L}$$

$$V = d_{xz} \cdot \frac{F_y}{\varepsilon \cdot W}$$

$$V = d_{xx} \cdot F_x \cdot \frac{t}{\varepsilon \cdot L \cdot W}$$

Piezoelectric materials

unit check: d*V = (coul/newt)*volt = (C/N)*(Ncm/C) = cm

material	piezoelectric constants (10^{-12} C/N)	relative permittivity	d/ε_r	Young's modulus (GPa)
quartz (thin films difficult)	$d_{xx} = 2.31$	4.5	0.5	107
polyvinyledene-fluoride (PVDF)	$d_{xz} = 23$ $d_{xx} = -33$	12	1.9 -2.75	3
$LiNbO_3$ (thin films difficult)	$d_{xz} = -4$ $d_{xx} = 23$	28	-0.14 0.82	245
$BaTiO_3$ (thin films difficult)	$d_{xz} = 78$ $d_{xx} = 190$	1700	0.04 0.1	
lead zirconate titanate (PZT)	$d_{xz} = -171$ $d_{xx} = 370$	1700	-0.1 0.2	53
ZnO	$d_{xz} = 5.2$ $d_{xx} = 246$	1400	0.003 0.17	123

Piezoresistive effects

- again fundamental origin is distortion of crystal structure
  - for conducting material can be viewed as distortion of bands
    - leads to change in effective mass
    - leads to change in resistivity / conductivity
  - is dependent on direction of stress σ relative to resistor axis and crystal directions

$$\frac{\delta\rho}{\rho} = \pi_{||} \cdot \sigma_{||} + \pi_{\perp} \cdot \sigma_{\perp}$$

- for silicon
  - depends on doping type and concentration, as well as orientation of wafer and layout of resistor
    - drops rapidly for doping > 10^{18} cm^{-3}

Piezoresitive coefficients for (100) Si, n, p < 10^{18}

doping type	$\pi_{\|\|}$ (10^{-13} m^2/N)	π_{perp} (10^{-13} m^2/N)	orientation of resistor
p-type	0	0	<100>
p-type	72	-65	<110>
n-type	-102	53	<100>
n-type	-32	0	in <110>

- also depends on temperature
  - ~ 0.25% per °C

Strain gauge

- **frequently "gauge factor" used instead of piezo-resistive coefficients**

$$GF = \frac{\delta R / R}{\underbrace{\delta L / L}_{\text{strain}}}$$

- **consider simple bar**

$$R = \frac{\rho \cdot L}{A} \quad \Rightarrow \quad dR = \frac{\rho}{A} \cdot dL + \frac{L}{A} \cdot d\rho - \frac{\rho \cdot L}{A^2} \cdot dA$$

$$\left(dR = \frac{\rho}{A} \cdot dL + \frac{L}{A} \cdot d\rho - \frac{\rho \cdot L}{A^2} \cdot dA \right) \cdot \frac{1}{R} \quad \Rightarrow \quad \frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{dA}{A}$$

- **assume cylindrical geometry**

$$A = \frac{\pi D^2}{4} \quad \Rightarrow \quad \frac{dA}{A} = \frac{2dD}{D}$$

- **recall Poisson's ratio**

$$\nu = -\frac{\delta D / D}{\delta L / L} \quad \Rightarrow \quad \frac{dA}{A} = -2 \cdot \nu \cdot \frac{dL}{L}$$

Strain gauge

- **so have**

$$\frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{dA}{A} \qquad \frac{dA}{A} = -2\cdot\nu\cdot\frac{dL}{L} \quad \Rightarrow \quad \frac{dR}{R} = (1 + 2\cdot\nu)\frac{dL}{L} + \frac{d\rho}{\rho}$$

- **so the gauge factor is now**

$$GF = \frac{\delta R/R}{\underbrace{\delta L/L}_{\text{strain}}} = \frac{\delta R/R}{\varepsilon} = (1 + 2\cdot\nu) + \frac{\delta\rho/\rho}{\varepsilon}$$

- **in terms of the piezoelectric coefficient:** $\frac{\delta\rho}{\rho} = \pi_{||}\cdot\sigma_{||}$

- **and recall relation between stress and strain:** $E\cdot\underbrace{\varepsilon}_{\text{strain}} = \underbrace{\sigma}_{\text{stress}}$

- **so finally** $GF = (1 + 2\cdot\nu) + \frac{\pi_{||}\cdot\sigma}{\varepsilon} = (1 + 2\cdot\nu) + \pi_{||}\cdot E$

Thermo-electric effects

- Seebeck effect
  - temperature gradient across a conductor gives rise to an electric field
    - tends to oppose charge flow that results from the temperature gradient
  - usually measured in reference to another material, forming a bi-metallic junction
    - usually referenced to platinum
    - most metals: ~5 μV/°C
    - silicon: depends on doping, ~ - 500 μV/°C for light doping
- Peltier effect
  - current flow across a junction between dissimilar materials results in a heat flux across the junction
    - thermo-electric coolers

Electrostatic/magnetostatic forces

- simplest approach: energy method
  - recall that energy = force (vector) • "travel" (vector)
  - then

$$F = \frac{\partial(\text{energy})}{\partial(\text{distance})}$$

  - note that this can give the TOTAL force (not pressure) if you can identify a single spatial coordinate that is parallel to the force
  - simple example: parallel plates
    - electrostatic: applied voltage V
    - magnetostatic: current I