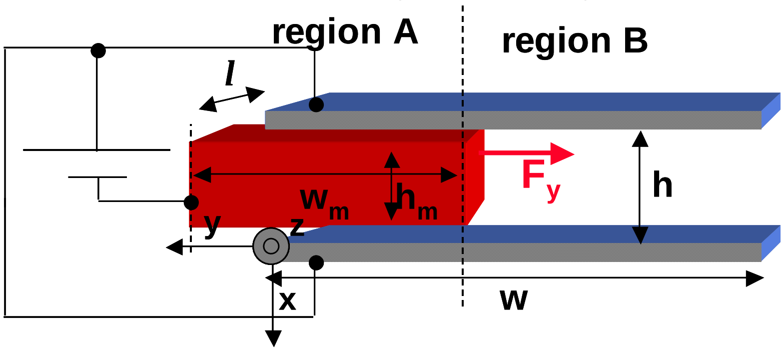
Homework set 3 Spring 2001
EE 397K ADV STDS IN ELECTRICAL ENGR; Unique Number 15170
date: 3/19/01
due: 3/28/01

1. Consider the "electrostatic" actuator shown below, made of two fixed metal plates and one metal slab that is free to move in the y direction only:



Using the simplest approximations (no fringe fields) we have for the force in the y direction:

$$F_y = \frac{\partial U_{cap}}{\partial y} = \frac{1}{2}\left[-\frac{\varepsilon_r \cdot \varepsilon_o \cdot l}{h - h_m}\right] \cdot V^2 \qquad (y > 0)$$

Assume the bias voltage V is constant, independent of time. Derive expressions for the velocity and position of the metallic slab as a function of time, assuming its initial position is such that y (t = 0) = w_m. Recall you'll just be applying Newton's Second Law $F = m \cdot a$, for the case of constant force, starting at zero initial velocity. Here NSL looks like:

$F = \rho \cdot l \cdot w_m \cdot h_m \cdot \frac{d^2 y}{dt^2}$, where ρ is the density of the metal slab; hence you need to solve

$$\frac{1}{2}\left[-\frac{\varepsilon_r \cdot \varepsilon_o \cdot l}{h - h_m}\right] \cdot V^2 = \rho \cdot l \cdot w_m \cdot h_m \cdot \frac{d^2 y}{dt^2} \qquad .$$

The minus sign just means the direction of the force is in the $-\hat{y}$ direction.

Now that you know y(t), you also know the capacitance as a function of time. Recall that the current flowing in the circuit is just dQ/dt, or $I(t) = \frac{\partial C}{\partial t} \cdot V + \frac{\partial V}{\partial t} \cdot C$. In this case V is constant, so find I(t).

Let's actually do the numbers for two geometries:

a) Take V = 10 volts, and assume that w = *l* = 1cm, w_m = 0.5cm, h = 0.1cm, h_m = 0.09cm, and ρ_m = 3grams/cm^3. How long does it take for the slider to completely enter the plates? Find the final velocity of the metal slider, and the current flowing in the circuit as a function of time.
b) Take V = 10 volts, and assume that w = *l* = 100µm, w_m = 50µm, h = 10µm, h_m = 9µm, and ρ_m = 3grams/cm^3. How long does it take for the slider to completely enter the plates? Find the final velocity of the metal slider, and the current flowing in the circuit as a function of time.